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THE DEMAGNETIZING FACTORS FOR CYLINDRICAL IRON RODS.

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Outline of the Subject.

It has long been known that when an unmagnetized iron bar is placed in a fixed magnetic field H' and thereby becomes magnetized, the actual force H within the iron is not so great as the original permanent magnetic force at the same point before the iron was introduced. The vector difference H_i, between the original force and the actual force resulting after the iron is brought in, is called the "demagnetizing force" due to the magnetism which has been induced in the iron. An original uniform field does not in general induce a uniform demagnetizing field within a piece of iron; in fact, it is commonly accepted that there is only one practical exceptional case: an iron ellipsoid placed so that a given one of its axes is parallel to the direction of the original uniform field. In this case the demagnetizing force for a given ellipsoid with a given axis parallel to the field is simply proportional to the resulting uniform intensity of magnetization I; and the proportionality-factor N is found by theory to depend only on the dimensions of the ellipsoid, that is on the semi-axes a, b, and c. Moreover, when the ellipsoid is a body of revolution, so that $b = c$, then we have a simple formula expressing N as depending solely on the value of the ratio a/b. This N is commonly called the "demagnetizing factor" for the ellipsoid.

Lord Rayleigh[1] first pointed out how from a knowledge of N a hysteresis curve obtained for an iron ellipsoid of revolution and plotted on the B vs. H' plane, could be "sheared back" into the limiting hysteresis curve for an ellipsoid of the same cross-section, which would be approached as the length of the axis which lies parallel to the field grows longer and longer. The same process is evidently applicable to a simple magnetization curve obtained by letting the applied field H'

[1] Phil. Mag., **22**, 175–183 (1886).

range from 0 to its maximum value, increasing continuously, and the iron being initially unmagnetized. The curve obtained by back-shearing is called the "normal" curve of magnetization for the kind of iron used. As the applied field H' is now the same as the resulting field H, the demagnetizing field having disappeared, this normal curve gives us the true permeability μ and susceptibility κ for every H, and is therefore the characteristic curve of the iron which we must use in order to get correct values for the physical quantities mentioned. Ewing and other investigators have made much use of this back-shearing process in working out hysteresis curves obtained for long iron wires, it being assumed, while experimental determinations were still lacking, that cylindrical iron wires could be regarded as behaving magnetically like ellipsoids of the same length and cross-section, provided the ratio of length to diameter was not too small.

The first attempt to find numerical values for the demagnetizing effect in cylindrical iron rods was made in 1894 by Du Bois[2] in discussing the only magnetization curves with varying length of rods which had up to that time been published: six by Ewing, obtained ballistically,[3] and a few by Tanakadaté, taken by a magnetometric method.[4] From these results Du Bois constructed a table of values for N for values of $\mathfrak{m}$ ranging from 10 to 1000, where $\mathfrak{m}$ = ratio of length L to the diameter D, of the rod. He evidently considered that N remains practically constant for the whole range of magnetic intensity. Du Bois's values of N for cylinders are from 10 per cent to 20 per cent smaller than for the corresponding ellipsoids, that is ellipsoids having the same ratio of length to maximum cross-section.

In 1895 C. R. Mann published[5] an extended series of results, obtained magnetometrically, for the demagnetizing factors of iron cylinders. The leading points brought out by this investigator, for the rods experimented on, most of which were of small diameter, are: (1) The N's for cylinders are very nearly constant for all intensities of magnetization below $I = 800$; after this point they increase rapidly as I increases. (2) For the range in which the N's are practically constant, they vary but a very few per cent from the values of the N's for the corresponding ellipsoids. Mann does not believe that ballistic and magnetometric determinations of N will give comparable results.

The most recent work on the demagnetizing factor which I have seen, is embodied in a short but extremely suggestive paper published

[2] Magnetische Kreise, Berlin, 1894, pp. 36–45; Wied. Ann., **46**, 485–499 (1892).
[3] Phil. Trans., **176**, II, 535 (1885).
[4] Phil. Mag., **26**, 450 (1888).
[5] Dissert., Berlin, 1895; Phys. Rev., **3**, 359–369 (1896).

in 1901 by Carl Benedicks.[6] This investigator, while working on the subject of pole-distances in cylindrical rods, interested himself in a few careful experiments on the demagnetizing factors. He gets for a hard steel rod of diameter 0.8 cm. and a length equal to 25 diameters, hysteresis curves by means of both the magnetometric and the ballistic methods. Then by turning it down on the lathe, he transforms the same specimen of iron into an ellipsoid of revolution of length equal to 30 diameters, and gets a hysteresis curve magnetometrically. This last curve is, by means of the known ellipsoid N for $\mathfrak{m} = 30$, back-sheared into the "normal" curve, which, according to Benedicks, can then be used to determine the N for any point on either the ballistic or the magnetometric curve for the cylinder. The result is that the magnetometric N behaves qualitatively exactly like that of Mann, but the ballistic N, after likewise remaining practically constant up to $I = 800$, *decreases* rapidly as I is further increased.

The present paper is an attempt to contribute to the subject a discussion of the demagnetizing factor for cylinders as determined ballistically. It will appear later that the curve on the B vs. H' plane (or the I vs. H' plane) which determines the back-shearing from a magnetization curve of a finite cylinder to the limiting normal curve, is quite different from the straight line which obtains in the case of the ellipsoid of revolution. It has, in fact, two opposite curvatures: one near the origin, and the other soon after the maximum value of the susceptibility has been passed. The first curvature is not very marked, and it turns out, as has been found before for the magnetometric N, that up to values of $B = 10{,}000$ (or $I = 800$) the ballistic N is not far from constant. The upper part of the curve, however, has a violent turn toward the B-axis (or I-axis) just as has been observed by Benedicks for his short steel cylinder. Theoretical reasons can be given to account in a general qualitative way for these experimental results.

Hitherto it has been the common custom, for lack of experimental evidence on the subject, to regard the N for iron cylinders, leaving out of consideration the variation of this coefficient with the I, as depending only on the ratio $\mathfrak{m} = L/D$, and not on the absolute dimensions of the rod. As practically all the previous results have been obtained from experiments on iron cylinders having a diameter of less than 1 mm., that is, mere iron wires, the question has naturally not received any attention. In the present work the writer had at his

[6] Bih. Svenska Vet.-Akad. Handlingar, **27**, (1), No. 4, 14 pp. (1902); Wied. Ann., **6**, 726–761 (1901).

disposal two magnetizing solenoids very much longer than any which have ever been used before, as far as he knows. Thus it was made possible to obtain complete series of magnetization curves, yielding tables of values for N, for a large number of iron rods, ranging in diameter from 0.2381 cm. to 1.905 cms. The results disclose quite a marked dependence of N on the D, the L/D and I being considered constant. In fact the general rule may be stated that the value of N decreases as the diameter of the iron rod increases.

In the work both the "reversal" and the "step-by-step" methods have been used, and the results obtained may be interesting to some who have had occasion to observe the peculiar disagreements in the results given by these two methods. As a rule the N's calculated from reversal curves will be smaller than those obtained from the "step-by-step" method under the same conditions.

Introduction.

When a piece of homogeneous isotropic soft iron of any shape is placed in a magnetic field, it will always become magnetized, and the induced magnetism will in general show its existence by changing the original field outside the iron. The only exceptional cases are those in which the iron is "endless," that is, it is in the form of an anchor ring or a rod of infinite length, with the magnetizing solenoid wound directly over the iron. Whenever an apparent magnetic distribution of superficial charge σ and volume charge ρ is induced by polarization on or in any body of iron, the magnetic field H_i due to it combines with the magnetizing field H' to give a resultant field H, so that the actual field which determines the intensity of magnetization I is given at every point by the vector equation

$$H = H' + H_i;$$

and $I = \kappa H$, where $\kappa =$ susceptibility of the iron. Outside the iron H will usually be less than H' in some portions of space, and in others it will be greater than H'. But inside the iron H will in general, perhaps always, be less than H'. Thus in the case of a sphere of soft iron placed in a uniform field H', we shall have, from the theory given in most of the text-books on electricity and magnetism,[7] a uniform field of intensity $H = H' - \frac{4\pi}{3} I$ within the sphere at any point A, while the

[7] Maxwell, II, §§ 437–438; Webster's Electricity and Magnetism, p. 371; Peirce's Newtonian Potential Function, p. 205.

intensity is $H' - \frac{4\pi}{3}I + 4\pi I$ at the point B just outside the sphere on that line of H' which passes through the centre of the sphere, while at all points C just outside the sphere and lying in a plane passing through the centre of the sphere and perpendicular to the H'-line mentioned, the intensity will be $H' - \frac{4\pi}{3}I$. Figure 1, reproduced from Figure 76 on page 373 of Webster's "Theory of Electricity and Magnetism," shows the resultant lines of force in this case. For a ring or an infinite rod of constant cross-section with the magnetizing solenoids properly arranged, we should get $H_i = 0$, and $H = H'$.

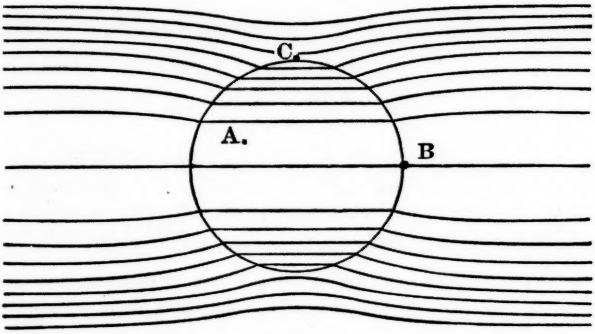

FIGURE 1.

A sphere of permeability 3 in a uniform magnetic field.

At any point in an iron body subjected to a magnetizing field H', the strength of the field H_i can be regarded as a function of I. If in particular we write the scalar equation

$$H_i = NI$$

and remember that in practical cases the H_i is a field opposed to H', or tending to demagnetize the iron, then we may speak of the factor N as the "demagnetizing factor" of the particular body of iron at the point considered, with reference to the permanent magnetizing field used, which in all practical cases will be a uniform one. Since H_i is in general an unknown function of I, therefore N is also some function of I. As the H_i in the cases to be considered will be directed exactly oppositely to H' in that part of the iron which we shall be interested

in, we shall hereafter use the scalar values for H', H_i, and I, so that our first equation will become

$$H = H' - H_i = H' - NI.$$

The only case of a magnetized body not endless, in which we can always calculate what the H_i will be, is where an iron ellipsoid is placed with one of its axes parallel to a uniform magnetizing field H'. If the equation of the ellipsoid is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1,$$

then it is shown in text-books on the mathematical theory of electricity and magnetism,[8] that if there exists on the ellipsoid a surface distribution of magnetic matter everywhere equal to

$$\sigma = I \cdot \cos(x, n)$$

where I is a constant, and (x, n) is the angle between the positive direction of the x-axis and the exterior normal to the ellipsoid, the volume density ρ being zero throughout the ellipsoid, then the magnetic field due to this distribution is constant at every point within the ellipsoid and equal to

$$H_i = 2\,\pi abcIK_0,$$

where

$$K_0 = \int_0^\infty \frac{ds}{(s+a)^{\frac{3}{2}}(s+b)^{\frac{1}{2}}(s+c)^{\frac{1}{2}}}.$$

This field H_i is directed parallel to the negative direction of the x-axis, and tends to demagnetize the iron; we see furthermore that it is directly proportional to I. The constant I is simply the intensity of magnetization, uniform within the ellipsoid. To keep this magnetic distribution in equilibrium it is sufficient if we apply a uniform magnetic field parallel to the positive x-axis, of such a strength H', that when diminished by the demagnetizing field H_i, there will remain in the ellipsoid the uniform resultant field $H = I/\kappa$, where κ is the susceptibility corresponding to the magnetization I, for the kind of iron under consideration. Of course if the σ has initially been chosen greater than the maximum value of magnetic intensity attainable, it will be

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[8] Maxwell, II, §§ 437 and 438; Webster, Elec. and Mag., §§ 192, 196; Peirce, Newtonian Potential Function, § 69.

impossible to realize such a distribution. If we have a possible case, then

$$H = H' - H_i = H' - 2\pi abc \cdot K_0 \cdot I.$$

Now the factor $2\pi abc \cdot K_0$ is constant for a given ellipsoid, and is called its "demagnetizing factor" N. When the iron is an ellipsoid of revolution ($b = c$), we can integrate K_0 and get a simple formula for N as a function of a/b, the ratio of the length of the ellipsoid to its greatest diameter.[9] It is, when written in terms of $\mathfrak{m}$,

$$N = \frac{2\pi\mathfrak{m}}{(\mathfrak{m}^2 - 1)^{\frac{3}{2}}} \log\left(2\mathfrak{m}\sqrt{\mathfrak{m}^2 - 1} + 2\mathfrak{m}^2 - 1\right) - \frac{4\pi}{\mathfrak{m}^2 - 1}.$$

When 1 is negligible in comparison with $\mathfrak{m}^2$ the formula assumes the simple form

$$N = \frac{4\pi}{\mathfrak{m}^2}(\log 2\mathfrak{m} - 1).$$

This N does not depend, therefore, on the softness of the iron nor on the magnetizing field, provided the iron ellipsoid was initially demagnetized and our magnetizing field has been continuously increased from zero to its final value.

If the iron is perfectly "soft," or incapable of retaining magnetism when the magnetizing force H' is withdrawn, then any field H' will produce a unique magnetization. The uniform H' along the major axis of the ellipsoid of revolution will therefore produce such a magnetization as we found would be kept in equilibrium by the same H'. As the iron we deal with in practice is not "soft," but shows hysteresis, we find it necessary to define susceptibility as the ratio of I/H when the iron is *slowly* carried from zero magnetization to the value I, the magnetizing field to increase slowly and continuously up to the proper value H'. Under these conditions it is reasonable to suppose that any magnetizing field will give a unique magnetic distribution, and our results hold true.

Suppose we desire to measure the susceptibility of a specimen of iron in accordance with our ideal definition, so that it may be free from ambiguity; let us consider the suitability for this purpose of the various experimental methods now in use. The fluxmeter is an instrument recently invented, which attempts to give permanent deflections which are proportional to the changes of magnetic induction through a secondary circuit, and these deflections are independent of the time-

[9] Maxwell, II, §§ 437–438.

intervals in which these changes complete themselves. The performance of this instrument is as yet far from satisfactory. If it could be made perfect, we should have an ideal method for permeability determinations, for we could then increase the magnetizing field as slowly as we please, reading off the corresponding magnetic inductions for any desired values of the field. It is probable that the oscillograph methods are at present much more to be preferred, as they can be made to record accurately the slow and long-continued changes of magnetic induction through large masses of iron.

A very good method to use is the "step-by-step" magnetization, where ballistic throws are produced in a Thomson galvanometer, or in a D'Arsonval galvanometer when we use proper precautions to secure the proportionality of throws to the flux changes. These changes in magnetic induction through a secondary coil wound around the iron specimen to be tested are most conveniently obtained by sudden decreases (or increases) in the resistance of the primary circuit, consisting usually of a storage battery and the magnetizing solenoid. By this arrangement it is not difficult to obtain cyclic hysteresis curves. It has been shown[10] that the maximal induction B (or I) which is reached varies with the number of steps taken, the difference being most marked in the region of greatest permeability. As the number of steps is increased continually in different experiments, the B vs. H curves move nearer the H'-axis, but soon approach the limiting curve for a slow continuous change of H', which, as we saw before, is the one curve that, after the proper back-shearing, will give values for the permeability (and susceptibility) conformable to the ideal definition. Lastly in order of accordance with the ideal definition of susceptibility comes the "reversal" method of measuring ballistic induction throws, which is entirely contrary to a slow magnetization, but which is often the most convenient of all the methods to use, and which gives the most self-consistent determinations; that is, repeated magnetizations will give almost identical results. Both the "step-by-step" and the "reversal" methods of measuring magnetic induction may give results depending on the particular experimental conditions employed, unless one takes proper precautions. Thus the time-constant L/R of the primary circuit should be only one or two per cent of the time it takes the galvanometer-needle to reach its greatest deflection, which time will be the quarter-period of the needle suspension system. It should be noted that when there is a great bulk of iron in the mag-

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[10] F. Rücker, Diss. Halle, 1905, 106 pp. 20 plates; Elektr. ZS. **26**, 904–905, 979 (1905).

netizing solenoid, the L may be enormously large. There are two ways of realizing the condition of the smallness of the time-constant as compared with the quarter-period: (1) We may use a storage battery of high E.M.F. in the primary circuit, which will necessitate large R's in the circuit in order to give magnetizing fields of the desired intensity; (2) It is quite possible to increase the moment of inertia of the needle-suspension so as to give a complete period of several minutes. Several of the experimental series obtained in this investigation by means of the reversal and step methods illustrate very forcibly how these two different methods may lead to various determinations of the susceptibility. Finally, the magnetometric methods are often very useful, especially in accurate determinations of magnetic moment of short iron magnets. With none of these magnetometric methods can we measure the I at any particular part of the iron bar, but get instead a mean value of I (moment/volume of bar) for the whole rod. Plotting I vs. H' curves for various lengths of soft iron cylinders, we can find mean demagnetizing factors N, by means of which a "normal" curve can be constructed. But it will be seen, after a little reflection, that the curve Mean I vs. Mean H which we get here is not necessarily the same, or even approximately the same, as the "normal" curve of I vs. H, which gives corresponding values of I and H in the middle of the bar immediately surrounded by the secondary coil, and which may be regarded as an extremely close approximation to the I and H at a single point in the iron. It is this fact which accounts for the wide difference which has been found between the N as determined ballistically and the N as determined magnetometrically. It is hardly likely that the process of back-shearing a magnetometric magnetization curve will yield a curve from which anything like the true susceptibilty can be found.

Returning now to our iron ellipsoids of revolution, we see that if we know the ratio of the length to the diameter of one of them, we can calculate exactly what the demagnetizing factor N will be. Ewing and Du Bois, in their texts on magnetism, give tables of values of N (see page 204) for various ratios a/b. It follows from a paper by Lord Rayleigh,[11] that if we magnetize any iron ellipsoid of revolution having a known ratio a/b, from zero magnetism to full saturation, measuring the I ballistically by means of a small secondary coil around the middle part of the rod, and plot out the curve I vs. H', we can "back-shear" this curve parallel to the H'-axis by the amount $H_i = \Delta H = NI$, and thus construct the "normal magnetization" curve, for which $H = H'$, and from which alone the true susceptibility can be found for every I.

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[11] Phil. Mag., **22**, 175–183 (1886).

Suppose now that we have any elongated piece of iron with a secondary coil wound around it near the middle and connecting with the terminals of a ballistic galvanometer. Suppose also that the normal magnetization curve for the kind of iron used were known, say, by taking measurements ballistically on an anchor-ring made of the same material. (As a matter of fact this method does not apply, for by welding the ends of a rod together to form a ring, we change the magnetic behavior of the iron unavoidably, to say nothing of differences which exist in two different specimens of iron made from the same kind of iron.) If we now find experimentally the actual magnetization curve, and plot it together with the normal curve on the I vs. H' plane, and plot on a similar plane, which we shall call the I vs. $(H'-H)$ or the I vs. H_i plane, the differences of the abscissae (which are $\Delta H = H_i = NI$) of the two curves for each I, against this same I, we shall call this last curve the "N-curve" for the particular piece of iron and the particular position of the secondary coil, it being understood that we have placed the iron in a definite position in a given magnetic field, or distribution of lines. The I of the actual magnetization curve is the average I existing in the volume of iron immediately surrounded by the windings of the coil. In general we do not know what the form of the N-curve may turn out to be, until we obtain it experimentally; in the ellipsoid of revolution placed with its major axis parallel to the uniform field, this N-curve will, according to theory, obviously be a straight line through the origin and making with the I-axis the angle whose tangent is equal to $N \cdot$(ratio of H' scale unit to I scale unit).

Now since ellipsoids of revolution are not very easily constructed, the case most important for magnetic measurements in laboratory practice is that of the cylindrical iron rod with ends squared off, and the secondary coil wound around just in the middle part of the rod, a uniform magnetizing field, such as can be secured inside a long solenoid, being used to produce the H'. Here we do not obtain a uniform I by placing the rod in a uniform field, and although the problem is determinate mathematically, no one has as yet succeeded in obtaining the solution. The great difficulty lies in the fact that the susceptibility is not constant throughout the rod for any given H'. The lines of magnetization run parallel only through the middle cross-section of the rod, where the secondary coil is wound. If, then, we wish to know the N-curves for some kind of iron in the form of cylindrical rods, our only resource is to find experimentally a series of I vs. H' curves for greater and greater values of $\mathfrak{m} = L/D$, where L = length, and D = diameter of the rod. Then we must find, by some extrapolation method, or otherwise, the limiting curve as $\mathfrak{m}$ becomes larger and

larger. We may then plot out the abscissa-differences between this normal curve and all the others, and thus actually construct the N-curves.

The only experimental magnetization curves for a number of varying $\mathfrak{m}$'s which had been published before 1895 are those obtained by

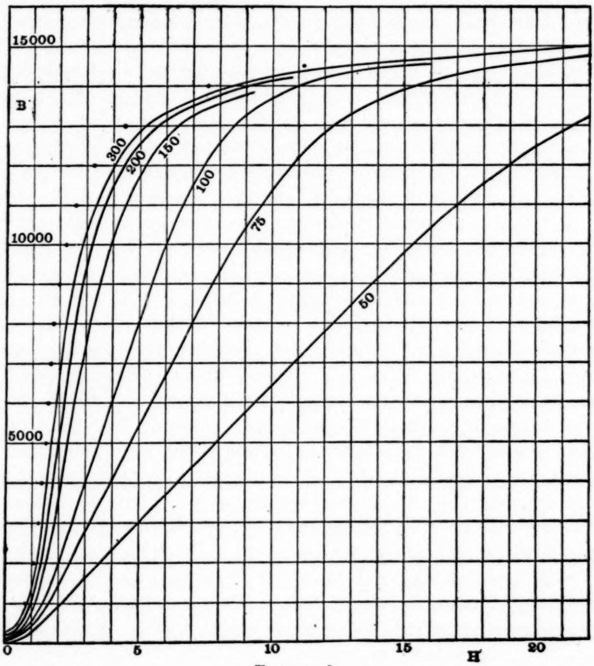

FIGURE 2.

Ewing's magnetization curves for a soft iron wire of diameter 0.158 cm.

Ewing [12] for $\mathfrak{m} = 50, 75, 100, 150, 200$, and 300 (see Figure 2), and some by Tanakadaté for rather small values of $\mathfrak{m}$, his highest being about $\mathfrak{m} = 39$. Ewing's iron cylinder was a wire of diameter $= 0.158$ cm. and original length $= 47.5$ cms., the other $\mathfrak{m}$'s being obtained by cutting off pieces from each end. The maximum permeability for this iron was found to be $\mu = 3500$. Tanakadaté's iron wires were of

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[12] Phil. Trans., **176**, II, 535 (1885).

diameter = 0.153 cm., the length varying from 2 to 6 cms., also of diameter = 0.115 cm. and a length originally 33.4 cms. For the shorter specimens he used Gauss's A position, that is, the rod is placed east and west and the magnetometer is placed in the prolongation of the rod's axis; for the longer wires Ewing's method was used, in which the solenoid and wire are placed vertically, with an extra solenoid to compensate for the earth's field, and the magnetometer being placed east or west of one end of the wire.

Du Bois subjected these data to a very extensive discussion. He developed the proposition that, provided the length of the rod is sufficiently great compared with its diameter, then $N\mathfrak{m}^2 =$ constant. This constant he finds from Ewing's curves to be equal to 45, provided $\mathfrak{m} \geqq 100$. The reason why this formula cannot possibly hold for short rods is that the theory of Du Bois assumes that the average magnetization intensity I in the whole rod differs but very little from the I within the secondary coil in the middle of the rod; in other words, that the magnetization is practically uniform. Of course this is never realized for finite rods and ordinary fields H', but it seems at first sight as if the magnetization in a rod of large $\mathfrak{m}$ should be fairly uniform. If we follow Du Bois's method, which gave him the necessary data to construct his table of values for N in case of cylinders, we may measure abscissa-differences, which are proportional to N, for the curves for rods of large $\mathfrak{m}$'s, and form three or four simultaneous equations, each of which linearly contains x, the abscissa-difference of the normal curve and the I vs. H' curve for the largest $\mathfrak{m}$ used in the equations. Any two of these equations give x, and we can thus construct the normal curve, which gives us immediately all the N-curves by plotting abscissa-differences as before. Du Bois, from the meagre data at his command, found values for N for various $\mathfrak{m}$'s and has collected the results in tabular form (see table, page 204) in his book "Die Magnetischen Kreise in Theorie und Praxis" ("The Magnetic Circuit in Theory and Practice," translated by Atkinson). He apparently considers the N-curves to straight lines, as far as practical purposes are concerned, that is N is not a function of H (or I); at any rate he does not mention any such variation of N. And as to the question whether or not the N for a given $\mathfrak{m}$ and I varies with the diameter of the rod, no data were at hand.

Now there is no reason to believe the N-curves for cylindrical rods of the same diameter to be straight lines; and since we know that the building up of magnetization, and perhaps even the final result, is very decidedly modified by the bulk of iron magnetized, it is quite likely that thick massive rods of iron really give different values for N from

those calculated by Du Bois for the "iron wires" used by Ewing and Tanakadaté. And, lastly, it is quite possible that the N may vary with the degree of softness and other physical characteristics of the iron magnetized. The present investigation was therefore undertaken to test as accurately as possible the true nature of the N-curves, whether they are really straight lines or not, and their possible variation with the diameter of the rod. Moreover, a table of values of N determined carefully by the ballistic method for thicker rods than has been done so far, would be quite useful in the practice of electrical engineering as, for instance, in the designing of dynamo machinery.

Before discussing the experimental results let us consider theoretically the N-curves for a given kind of iron and a given diameter, the length alone being varied. We shall attempt to show that this backshearing curve has two opposite curvatures. Let us suppose that we know the normal magnetization curve of our iron. We want to learn something about the nature of the N-curve for a cylindrical rod of homogeneous isotropic iron whose length is finite but otherwise arbitrary. All the facts which we need are these: (1) The I has a maximum value I_∞, which is reached asymptotically by increasing the magnetizing force H' indefinitely. (2) In any finite cylindrical iron rod, no matter how short, the lines of magnetization can apparently be made straight, or I made uniform, by applying an infinite H'. And whenever I/H, the susceptibility, has rather small values, then the condition of uniform I is with some approximation realized. (3) Although the normal curve and all other I vs. H' curves for rods of finite length do not run into the origin tangential to the H'-axis, they do make a very small angle with it. In other words, the susceptibility approaches a small value $\kappa = 15$, or thereabouts, as the H' decreases indefinitely.[13] (4) The normal curve has one, and only one, point of inflection.

With regard to the second part of (2) it might be noted that the nonuniformity of I in an iron cylinder placed parallel to the lines in a uniform magnetic field is measured in a rough way by the largeness of the ratio H_i/H, the demagnetizing force divided by the resulting force, at the point considered. Now $H_i = NI = N\kappa H$, so that this ratio is merely $N\kappa$. Therefore, if we suppose for the moment that N for a given finite rod is nearly constant for a considerable range of I, it follows that the magnetization will be the nearer to uniformity the smaller the susceptibility is.

Let us then consider the N-curve for a rod for which $\mathfrak{m} = \mathfrak{m}_1$, say.

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[13] C. Baur, Wied. Ann., **11**, 399 (1880). Lord Rayleigh, Phil. Mag., (5), **23**, 225–245 (1887).

In Figure 3 let P and Q be two points on the I vs. H' curve for $\mathfrak{m}_1$, where Q has the ordinate of the point of inflection Q_0, and P is any other point of the magnetization curve. Now suppose the rod were magnetized by an infinite H' to the maximum I_∞, so that all the $\pi a^2 I_\infty$ lines are straight and enter and leave the rod at the squared-off ends (a being the radius of the rod). In this case the distribution of magnetism which we may consider the cause of the demagnetizing force H_i, or ΔH, is wholly superficial, and as far away from the secondary coil, where I is measured, as possible, and it has a perfectly definite value ΔH_∞, say, which we lay off on the I vs. $(H'-H)$ plane, getting the point K, and we draw the line OK. We see now that if, as we in-

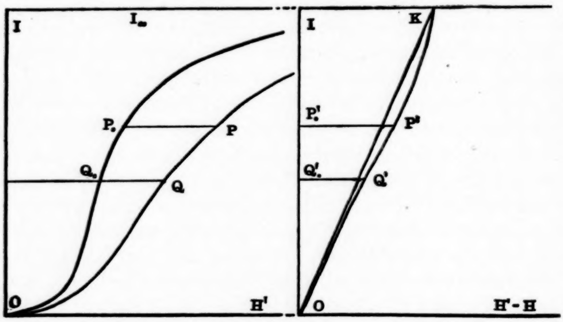

FIGURE 3.

Diagram illustrating magnetization and back-shearing curves.

crease I from zero to I_∞ by continually increasing H', the lines of magnetization were always straight, then the demagnetizing force would always be proportional to I, no matter what the susceptibility might be, and the N-curve would be the straight line OK. Another case where the N-curve would be a straight line OK_1 would be realized if the susceptibility were a constant for all values of I from O to I_∞. In this case no volume density would appear by magnetization, and any two fields H_1' and H_2', giving separately the surface densities of magnetism σ_1 and σ_2, could be superposed, so that a magnetizing field $H_1' + H_2'$ would give the superficial distribution $\sigma_1 + \sigma_2$. This last supposition would result in there being no limit to the intensity of magnetization. As a matter of fact the I is uniform only for an infinite H'. At the point P, if P is not the origin, more or less lines of induction will leave

the iron rod along the curved surface, as is well known. Now from the mathematical theory we know that in the case of "soft" iron B, or μH, is a solenoidal vector, continuous throughout all space, whether iron or air, not containing any fixed magnetic charges. Wherever lines of induction *leave* the surface of the iron we must therefore have positive σ; for the vectors H and I, although not solenoidal in the iron, have always the same distribution as the vector B, I is zero outside the iron, and $\sigma = I \cdot \cos(n, I)$. This means that a part of the surface distribution σ of the magnetism is closer to the middle of the rod than it would be if I were uniform. There is also some magnetic matter in the form of volume distribution ρ. This, however, does not materially influence the argument, although it complicates matters somewhat. We shall come back to the volume charge later. Therefore, as far as the surface magnetism is concerned, the demagnetizing force ΔH_P is for every point P actually greater than it would be if I were uniform. We thus reach the result that the N-curve has the end-points O and K, but lies everywhere else to the right of the straight line OK. Indeed for the most part the N-curve will be very decidedly to the right, for a very large number of the lines of induction will leave the iron rod before reaching the ends of the rod. The demagnetizing factor N_∞ is the minimum value of N, although ΔH_∞ is by no means vanishingly small. Near the origin the ratio of H to I is comparatively large, although of course still a fraction, so that according to (2) the I is more nearly uniform than for higher points on the curve, so long as we do not pass the point of maximum susceptibility, which is the point of tangency of a line drawn from the origin to the normal magnetization curve; therefore the N-curve is more nearly tangent to the line OK at the origin than for points a little more removed. As we increase H' from O to some point Q whose I is of the order of I at Q_0, the lines of magnetization increase continually, but a larger and larger fraction of lines leave the rod before reaching the ends, and N increases continually. Again, as we follow the magnetization curve from any very large but finite value of H' down toward Q, the I-lines spread out in greater and greater proportion, and the N increases for quite a long interval. This shows that the curvature of the N-curve changes sign at some point Q_1, which is a point of inflection for the N-curve, and probably the only one. We should expect, therefore, that the curve drawn in the second part of Figure 3 on the I vs. NI plane represents roughly the qualitative behavior of an N-curve for a finite rod.

It remains to be shown that the volume distribution does not invalidate the argument just given. From the theory of magnetism we know that this can be expressed in the form

$$\rho = \frac{h_\kappa h_V \cdot \cos(h_\kappa, h_V)}{\mu},$$

where $\mu =$ the permeability, h_κ and h_V the gradients of the susceptibility and resultant magnetic potential function, respectively, and (h_κ, h_V) is the angle made by the directions in which κ and V increase most rapidly. For we have by Poisson's Equation,

$$\nabla^2 V = -4\pi\rho,$$

and from the fundamental equation of magnetic polarization,

$$\begin{aligned}\rho &= -\text{ Divergence } I = -\left[\frac{\partial}{\partial x}(\kappa X) + \frac{\partial}{\partial y}(\kappa Y) + \frac{\partial}{\partial z}(\kappa Z)\right] \\ &= \frac{\partial}{\partial x}\left(\kappa\frac{\partial V}{\partial x}\right) + \frac{\partial}{\partial y}\left(\kappa\frac{\partial V}{\partial y}\right) + \frac{\partial}{\partial z}\left(\kappa\frac{\partial V}{\partial z}\right) \\ &= \kappa \cdot \nabla^2 V + \left[\frac{\partial \kappa}{\partial x}\cdot\frac{\partial V}{\partial x} + \frac{\partial \kappa}{\partial y}\cdot\frac{\partial V}{\partial y} + \frac{\partial \kappa}{\partial z}\cdot\frac{\partial V}{\partial z}\right].\end{aligned}$$

Eliminating the $\nabla^2 V$ we get the equation above. Now h_κ, h_V, and μ are all intrinsically positive. The h_κ becomes zero under special conditions, and is vanishingly small when the iron becomes fully saturated. Therefore the sine of σ is governed by the cos (h_κ, h_V) alone. Considering only the half of the iron cylinder on which the positive σ appears, we see that V always increases from the end of the rod toward the centre, while ρ does so as long as the magnetization at the centre of the rod has not been pushed beyond the maximum susceptibility point. Under these conditions (h_κ, h_V) is an acute angle, and therefore ρ is positive. Therefore the argument regarding the curvature of the N-curve in the neighborhood of the origin is even strengthened all the more on account of the positive ρ intensifying the demagnetizing force. Thus the lower curvature is proved (although not quite rigorously, mathematically speaking), and since the N-curve must end in the point K, there must be a curvature in the upper part of the N-curve directed oppositely to the first one.

An interesting fact perhaps worth noticing in regard to the volume distribution ρ of the magnetism is that as soon as the point of maximum susceptibility has been passed over, which will first occur at the centre of the rod, there will appear some *negative* ρ near the centre of the rod in that half of the rod which always carries the positive sur-

face distribution. This is due to the fact that (h_κ, h_V) now has become an angle of 180° at points in the axis of the rod and near the centre of the rod, while further away from the centre but still along the axis, where the κ has not yet reached its maximum, the angle (h_κ, h_V) is still zero. Somewhere between the two regions will be a curved surface for all points, of which κ has its maximum susceptibility, and h_κ is zero, and the angle (h_κ, h_V) is discontinuous by π, so that ρ is everywhere zero on the curved surface, which separates the regions of positive and negative ρ. As the iron is subjected to higher and higher fields H', this curved surface moves further and further away from the centre, until finally there is only negative ρ left in that half of the iron rod which has the positive surface magnetism. This occurs just as soon as every point in the iron has been magnetized past the point of maximum κ. The presence of this negative ρ may perhaps account very largely for the fact that N is not far from constant for quite a long range of I. When saturation of the iron with magnetism is approached more and more, the κ becomes nearly constant throughout the rod and continuously approaches zero, so that h_κ, and therefore the negative ρ, are both becoming vanishingly small. C. G. Lamb[14] gives a set of curves, reproduced in Figure 4, showing the variation of μ along an iron rod from centre to end for various applied fields, which illustrate the matter with perfect clearness. Of course the μ, when found, as Lamb did, by ballistic methods. with a search coil placed at varying distances from the centre, is the mean value of μ for the iron surrounded by the search coil, but it shows the variations along the rod very well indeed.

All the N-curves found in the experimental series of the present paper do not deviate to a very great extent from straight lines for values of B less than 10,000 or thereabouts. They show quite definitely the two curvatures which we were led to expect by theoretical considerations. Above this point, however, the N-curves have an ever-increasing tendency to turn to the left, and at last actually do move from right to left, so that finally we have not only the H_i/I $(= N)$ merely decreasing, but even the H_i decreasing. At first this was very puzzling, for it would seem natural to suppose that, although N must really decrease when the iron bar shows saturation, just as we were expecting from the theory, as long as more and more lines of magnetic induction are thrown into the rod when as yet unsaturated with magnetism, there is more and more magnetism induced, which ought to increase the demagnetizing field H_i continuously.

[14] Phil. Mag., (5), **48**, 262–271 (1899).

This, however, is not at all the case, and the actual facts emphasize the fallacy of considering the magnetization in long iron rods, *when not completely saturated*, as even approximately uniform. As will appear from the results obtained in this investigation, the values of N are not far from being constant below $B = 10{,}000$, and they are of the order of magnitude as those found by Du Bois from Ewing's curves, although always somewhat smaller. But let us now find what these N-values would be if our various rods were really uniformly magnetized. In other words, let us find the position of K of the straight line

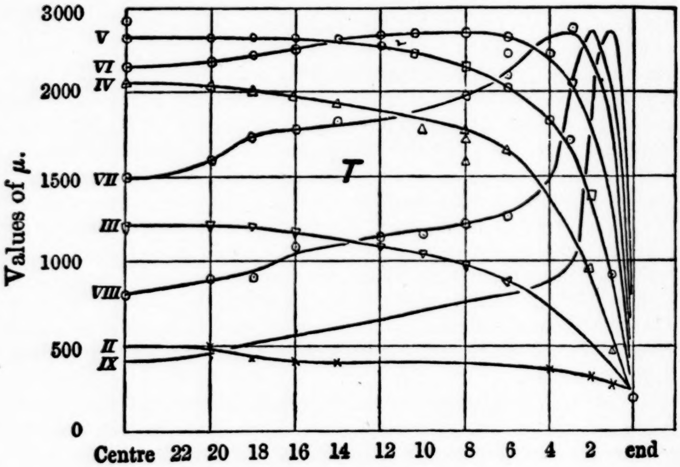

FIGURE 4.

Lamb's curves showing the change in permeability along an iron rod. The distances along bar are given in inches.

OK in Figure 3. Our rod has the length L and diameter D, so that uniform magnetization would mean $\pi\,(D/2)^2 I$ units of free positive magnetism on one end of the rod and the same number of negative units on the other end. If L is large compared to D, we may regard the demagnetizing field-intensity H_i (or NI) at the centre of the rod as caused by a single point-pole of strength $2\,\pi\left(\dfrac{D}{2}\right)^2 I$ at a distance of $L/2$ units of length from it. Then

$$H_i = NI = \frac{2\pi\left(\frac{D}{2}\right)^2 I}{\left(\frac{L}{2}\right)^2} = \frac{2\pi I}{\mathrm{m}^2}.$$

Therefore, for uniform magnetization,

$$N\mathrm{m}^2 = 2\pi = 6\cdot 28+.$$

This value for $N\mathrm{m}^2$, it will be noticed, is considerably less than the constant 45 as found by Du Bois from experimental data, and which constant led him to construct a table of values for N which, as we shall see later, is probably quite accurate for the iron wires of small diameter used by Ewing and Tanakadaté. Yet the conditions which Du Bois assumed in order that his theory might be applicable are precisely those which we have here assumed. For the shorter rods $N\mathrm{m}^2$ would be smaller yet, for the two reasons that the magnetism σ (or I) on the squared-off ends of the cylinder must now be considered further off than the distance $L/2$, and much of it acts at a small angle; of course the resultant H_i, which is now really given by a double integral, is directed along the axis of the rod. It is now clear that Figure 3 does not begin to show the tremendous sweep to the left, of the upper portion of the N-curve, which has been found by Benedicks[15] for his rod of steel where m was 25, and which really occurs in every one of the N-curves obtained ballistically.

Let us now compare the values of N for various ellipsoids of revolution, and those obtained by Du Bois for cylindrical rods, with the limiting values of N for uniform magnetization. The values for the shorter rods are calculated from the same formula as the longer ones.

The explanation of the great difference between the actual demagnetizing force under non-saturating fields and the demagnetizing force in case of uniform I is of course found in the fact that in the former case quite a large part of the lines of force leave the curved surface of the iron rod very near the middle of the rod, so that the contributions $\Delta M/r^2$ to the demagnetizing force count up very heavily in comparison with the magnetism nearer the end of the rod. An ideal uniformly magnetized rod of the same diameter, and having the same number of lines through its middle section as one which is actually magnetized in practice to less than saturation, must be only about $\sqrt{2\pi/45}$, or 0.374 times as long, if it is to produce as much demagne-

[15] Bih. Svenska Vet.-Akad. Handlingar., **27**, 1, No. 4, 14 pp. (1902); Wied. Ann., **6**, 726–751 (1901).

TABLE I.

DEMAGNETIZING FACTORS. (N.)

$m = L/D$ or a/b.	Ellipsoid.	Cylindrical Rods.	
		Du Bois.	Uniform I.
10	0.2549	0.2160	0.063
15	0.1350	0.1206	0.028
20	0.0848	0.0775	0.016
25	0.0579	0.0533	0.010
30	0.0432	0.0393	0.0070
40	0.0266	0.0238	0.0039
50	0.0181	0.0162	0.0025
60	0.0132	0.0118	0.0018
70	0.0101	0.0089	0.0013
80	0.0080	0.0069	0.00098
90	0.0065	0.0055	0.00078
100	0.0054	0.0045	0.00063
150	0.0026	0.0020	0.00028
200	0.0016	0.0011	0.000157
300	0.00075	0.00050	0.000070
400	0.00045	0.00028	0.000039

tizing force at the middle point of the rod as the other suffers. This induced magnetism (both σ and ρ) near the centre of a rod of iron magnetized to a value of B somewhat below 10,000, can be readily recognized by its effect on a small compass needle, which will be deflected the moment it is moved a few centimeters from the middle part of the rod toward either end.

It might be of interest to note that the highest possible demagnetizing force would be obtained by placing a very large slab of iron, with plane parallel faces, perpendicular to the lines of an infinite magnetizing field H'; the value of H_i would be $4\pi I_{\infty}$, when the slab is infinite in extent, but has any finite thickness. This H_i would, moreover, have

the same value at any point whatever in the iron slab. The value of N, the demagnetizing factor, is 4π throughout the slab. As in soft iron a negative force of H' less than 10 c.g.s. units of field intensity is sufficient to demagnetize the remanent magnetization which exists in the iron after the original magnetizing field is withdrawn, and the value of $4\pi I_\infty$ is about 200,000 of c.g.s. units, it is easily seen that on removing the infinite field the demagnetizing field H_i would instantly demagnetize the slab completely.

A diagram of the apparatus and its arrangement, as used practically throughout the present investigation, is shown in Figure 5.

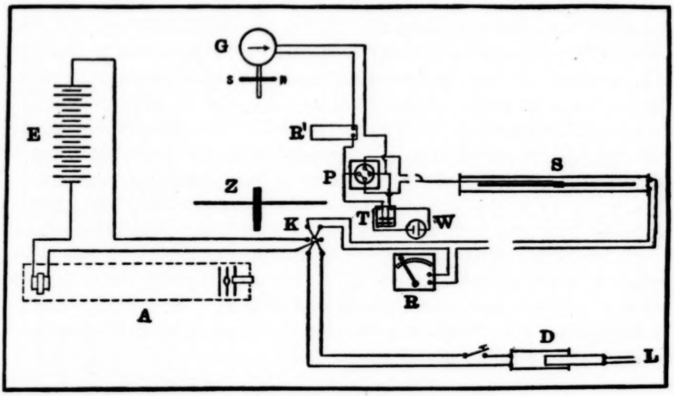

FIGURE 5.

Diagram of apparatus used in the Jefferson Physical Laboratory in obtaining magnetization curves for the present investigation.

EXPERIMENTAL METHODS AND APPARATUS.

G is a Thomson four-coil ballistic galvanometer with astaticised magnetic suspension, controlled by a permanent magnet S–N, and not shielded at all magnetically, for it was found that when shielded with three large cylindrical iron shells and heavy iron plate tops and bottom, certain unknown magnetic disturbances were caused in these shields, and effectually prevented the needle, which was then non-astatic, from coming to rest. E is the storage battery of from 5 to 20 cells, giving about 2 volts each, for furnishing the current in the primary coil. S is a large solenoid of the following dimensions:

Length = 207.7 cms.
Outside diameter = 5.97 cms.
Inside diameter = 3.63 cms.

This solenoid was wound on a tube of pasteboard with two wire coils of 3386 turns each, — of No. 18 wire, in six layers, — which were used in parallel, so that

$$H' = 4\pi nC/10 = 20.5 \cdot (\text{No. of amperes used}).$$

Later on in the work a still longer solenoid was built, in order to experiment on very thick iron rods. A is a "P–3" amperemeter, that is, one of the type so successfully used in the laboratory of the course Physics 3 in Harvard University; it reads with great accuracy up to 1.5 amperes. K is a double reversing knife switch, connected to the solenoid S, and also to a demagnetizing solenoid D, with an iron core in the small coil, which could be connected to the light circuit L. R is a rheostat in series with a system of variable resistance coils, to regulate the current. P is a reversing key to change direction of ballistic throw in the galvanometer. T is a tapping key arrangement with small battery, for bringing the galvanometer magnet needle to rest. Its circuit contains a very high resistance W. Z is the galvanometer scale with telescope, at 116 cms. distance from magnet system. R' is a resistance box in the secondary circuit; by varying this resistance the throws were kept under control, so as to give good accuracy in the readings.

The "P–3" galvanometer was frequently compared with a Weston milliamperemeter with shunt, and the sensitiveness of the galvanometer was often determined during the course of the work by charging a condenser of one microfarad capacity from a battery of four Samson (wet) cells whose voltage was read off on a voltmeter. The sensitiveness, given in centimeter divisions of throw per coulomb, ranged from 1.24 to 1.60. In the latter part of the work the condenser was charged by connecting across a standard resistance of 10 ohms, say, through which about 1 ampere was flowing, thus getting about 10 volts.

In the earlier half of the experiments the "reversal" method was used with great convenience and accuracy in the readings. The magnet suspension does not hold its zero very closely, but is slowly tossed about by magnetic disturbances over a range of 1 mm. scale reading, and sometimes more. Moreover, the zero position, which is quite definite at any one time, often changes slowly during the course

of the day. With the reversal method no attempt to read the zero was made, but instead a number of throws were taken alternately in the plus and minus directions, and then averaged. These throws often agreed regularly to about 1 part in 1000, when taken with a little care. The reversal method, however, has a possible error due to the time-constant of the primary circuit being comparatively large when there is much iron in the solenoid S, and also to the slow establishment of the magnetism in a thick iron rod. This was counterbalanced by making the complete period of the astatic system about 25 seconds, and finally 31 seconds.

The step-by-step method was used only in one series of experiments with the first solenoid S. This method is much harder to carry through successfully, especially since the battery E must maintain its voltage without appreciable drop while furnishing an increasing current for about half an hour, and the zero reading must be taken carefully every little while. Usually several curves were obtained for each length of the iron rod used, so that a good average curve could be constructed. As is well known, the two methods do not give the same magnetization curve, the one by the step method usually, but not always, lying below the reversal method curve.

The iron rods tested in the first solenoid were all of soft Bessemer steel, six feet long and of diameters ranging from 0.2381 cm. ($= \frac{3}{32}$ inch) to 1.270 cms. ($= \frac{1}{2}$ inch). The secondary coils consisted of from 30 to 400 turns of fine insulated wire wound directly over the middle of the rod. It was found necessary to reverse the magnetism about six times before reading the actual throws, otherwise the readings come out too low. After sufficient data had been collected to construct a curve, equal lengths of the rod were cut off from each end, so as to reduce $\mathfrak{m}$ from one value to the next. The ends of the rod were then filed smooth and plane. Then a curve was obtained for the shortened length of the rod.

After proper reduction of the observations, the magnetization curves B vs. H' were carefully constructed for all the $\mathfrak{m}$'s used, on a large sheet of millimeter paper of the dimensions 43 × 53 cms.

The next problem was to devise some means of getting at the normal curve ($\mathfrak{m} = \infty$). In the earlier part of the investigation frequent use was made of the principle which leads to Du Bois's experimental formula $N\mathfrak{m}^2 = 45$, when $\mathfrak{m} \geqq 100$. It was found that so long as B did not exceed the value 8000, the formula was fairly well satisfied for $\mathfrak{m} \geqq 150$, *provided* only one system of simultaneous equations was used. That is, supposing we had plotted out the actual magnetization curves for $\mathfrak{m} = 300$, 250, 200, and 150. If we take all these into

account, reckoning therefore the distance in any units of length, say millimeters, from the normal curve to the one for $\mathfrak{m} = 300$ as our unknown x, we shall find the whole set of equations giving a good average value for x, and thus we may construct what might be called "the normal curve based on $\mathfrak{m} = 300$." Now if we use only the curves for 250 to 150, so that our next x is the unknown distance from normal

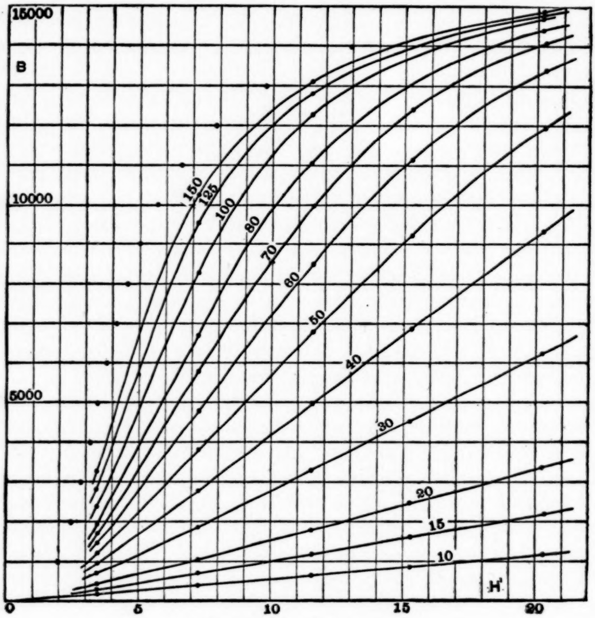

FIGURE 6. [TABLE II.]

Reversal magnetization curves for a Bessemer soft steel rod of diameter 0.6350 cm.

curve to the curve of 250, we shall again find values for x which satisfy all the equations moderately well. But the normal curve thus determined, which is the normal curve based on $\mathfrak{m} = 250$, will lie slightly to the right of the first one constructed, — at least every case tried gave this result. Similarly, the normal curve based on $\mathfrak{m} = 200$ will lie to the right of the one based on $\mathfrak{m} = 250$, and so for the one based on 175. For higher values of B than 8000 the formula fails to hold at

all. It should be noticed that as the iron rods become nearly saturated with magnetism, the magnetization curves bend around and become more and more parallel to the H'-axis, so that a very slight displacement of the curves up or down may result in proportionately large errors in the construction of the N-curves. The only thing to do is

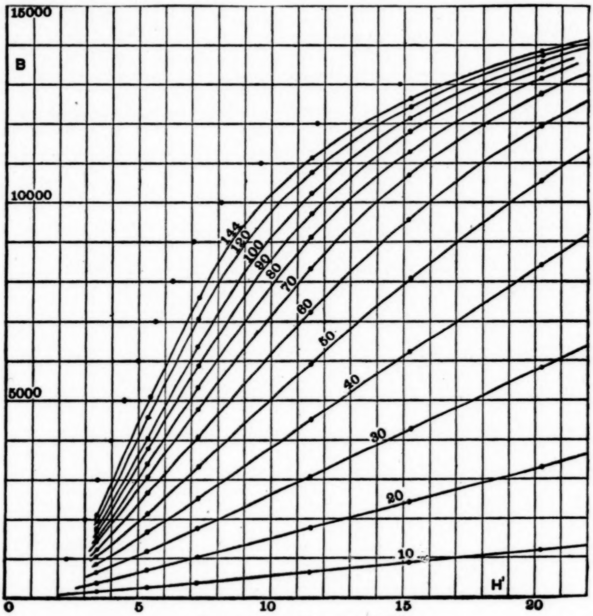

FIGURE 7. [TABLE III.]

Reversal magnetization curves for a Bessemer soft steel rod of diameter 1.270 cms.

to construct by "trial and error" methods a normal curve which will give the best possible results for the whole body of N-curves.

To be absolutely consistent the N-curves should be constructed from magnetization curves on the I vs. H' plane, for N is defined by $H = H' - NI$. Substituting in this the value for I from the fundamental equation $B = H + 4\pi I$, we get

$$H = H' - N\left(\frac{B - H}{4\pi}\right).$$

But as even for the high value $H = 30$, B is somewhere near 15,000, we see that the error introduced by neglecting the H in the brackets is but 1 part in 500, which is much less than the experimental errors. Therefore, since the ballistic throw is proportional to B, it is very

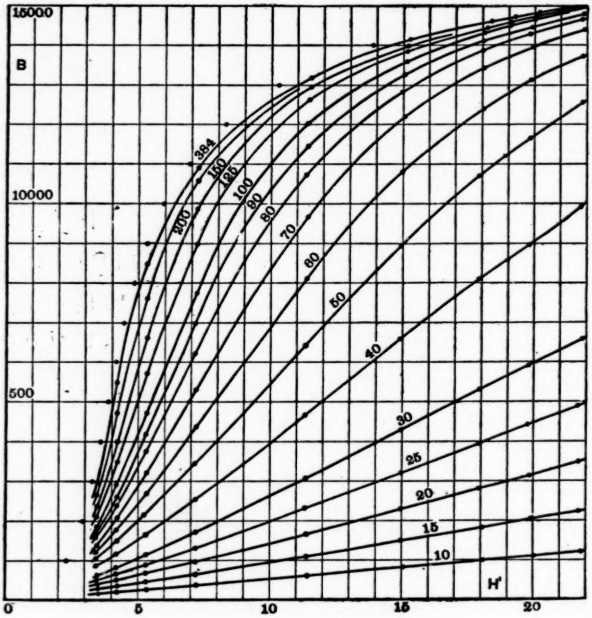

FIGURE 8. [TABLE IV.]

Reversal magnetization curves for a Bessemer soft steel rod of diameter 0.4763 cm.

much more convenient to construct the N-curves from the formula

$$H = H' - NB/4\pi.$$

EXPERIMENTAL RESULTS FOR DEMAGNETIZING FACTORS.

Let us now tabulate the actual values obtained for the end corrections, or demagnetizing factors N, of a number of rods of Bessemer steel (copper coated), which is a very homogeneous soft iron. Later on we shall see just how these values were determined, and give the

necessary data from which the most important table was constructed. It might be noted here that the results for the extremes of magnetization $B = 1000$, and $B = 12{,}000$ are somewhat less reliable, for reasons which will appear. The numbers 10 to 150 are the values of $\mathfrak{m}$ used.

TABLE II. [Figure 6.]

October 2, 1906.

Diam. = 0.6350 cm. = 1/4 in.

Reversals.

$B.$	Values of $N \times 10^4$.											
	$\mathfrak{m}=10$	15	20	30	40	50	60	70	80	100	125	150
1000	1990	1010	630	311	199	132	. .	. .	. .	. .	. .	. .
2000	. .	1028	644	328	199	137	104	79	64	41	. .	. .
3000	. .	. .	653	329	204	137	101	79	62	43	30	19
4000	. .	. .	. .	333	205	138	101	77	60	43	29	19
5000	. .	. .	. .	333	206	140	102	76	60	42	29	19
6000	. .	. .	. .	332	206	139	101	76	60	40	28	18
7000	. .	. .	. .	330	205	139	101	76	60	40	28	18
8000	. .	. .	. .	. .	205	139	101	76	58	39	26	17
9000	. .	. .	. .	. .	204	139	100	76	57	39	26	17
10000	. .	. .	. .	. .	202	137	99	75	56	38	25	17
11000	. .	. .	. .	. .	. .	134	97	73	55	36	24	17
12000	. .	. .	. .	. .	. .	132	95	70	53	34	22	16
13000	. .	. .	. .	. .	. .	. .	92	68	52	32	21	15
14000	. .	. .	. .	. .	. .	. .	86.	63	49	30	21	15
15000	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .

Below each value of $\mathfrak{m}$ is given the series of values of $N \cdot 10^4$ obtained, one for each interval of 1000 c. g. s. units of B, or gausses. Of course in all these experiments the column under the highest number $\mathfrak{m}$ gives values for the first curve obtained, for $\mathfrak{m}$ is always decreased by each sawing off of the ends of the iron rod.

See Figure 6 for the magnetization curves of October 2, 1906.

The normal curve as determined is indicated in all these figures by the dots spaced every 1000 units of B.

Figure 7 exhibits the curves taken on October 4, 1906, and shown in Table III. It will be seen that these curves are very much flatter than those of the $\frac{1}{4}$ in. rod and the $\frac{3}{16}$ in. rod which follows this one.

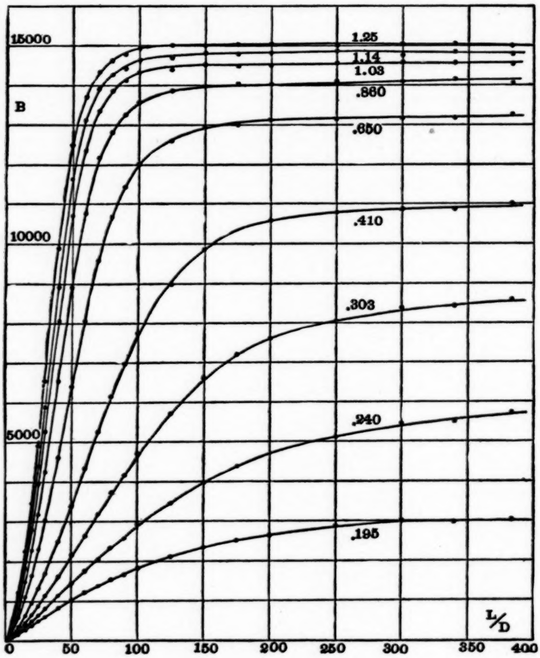

FIGURE 9. [TABLE IV.]

Curves showing variation of magnetic induction with different lengths of a Bessemer soft steel rod of diameter 0.4763 cm. The numbers affixed to the curves give the constant currents in amperes through the solenoid.

Figure 8 shows the original curves of October 9, 1906, and presented in Table IV.

From the data of these curves Figure 9 was also drawn. This shows the curves of constant current as the rod is increased in length. The numbers affixed to the curves give the current in amperes, so that the

applied field H' in the solenoid can be found by multiplying by the factor 20.5. It is seen that at first the induction increases very rapidly and nearly linearly. Then after a sharp bend the curve approaches a maximum induction asymptotically. It is interesting to see how for higher currents this maximum is reached very much sooner

TABLE III. [Figure 7.]

October 4, 1906.

Diam. = 1.270 cms. = 1/2 in.

Reversals.

B.	Values of $N \times 10^4$.											
	m = 10	20	30	40	50	60	70	80	90	100	120	144
1000	1820	590	300	190	126	95	. .	. .	. .	. .	. .	. .
2000	. .	614	317	198	135	97	74	62	50	42	31	23
3000	. .	635	325	203	137	99	76	63	50	42	31	23
4000	. .	. .	331	204	139	100	76	62	50	42	31	23
5000	. .	. .	331	204	139	100	76	62	50	41	30	23
6000	. .	. .	331	204	139	100	76	62	50	40	30	23
7000	. .	. .	. .	205	139	100	76	61	49	40	28	21
8000	. .	. .	. .	205	139	100	76	61	49	39	28	20
9000	. .	. .	. .	203	139	100	75	60	48	39	27	19
10000	. .	. .	. .	. .	137	99	73	59	48	38	27	19
11000	. .	. .	. .	. .	132	99	70	57	46	36	26	19
12000	. .	. .	. .	. .	123	90	66	54	42	33	24	18
13000	. .	. .	. .	. .	. .	. .	59	47	38	29	21	16
14000	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .
15000	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .

than for lower currents. As regards curvatures, the sharp bend, and approach to a maximum value, these curves bear a close resemblance to the magnetization curves, when plotted on the I vs. H' plane.

See Figure 10 for the magnetization curves accompanying Table V, October 20, 1906. These are also quite steep.

No figure is given for the results obtained on November 6, 1906, and collected in Table VI. The curves are very steep.

See Figure 11 for the magnetization curves corresponding to Tables VII and VIII, of November 16, 1906. The curves passing through the crosses are the ones obtained by using the method of steps, while the

TABLE IV. [Figure 8.]

October 9, 1906.

Diam. = 0.4763 cm. = 3/16 in.

Reversals.

B.	Values of $N \times 10^4$.															
	$m = 10$	15	20	25	30	40	50	60	70	80	90	100	125	150	175	200
1000	2001	1023	638	434	319	196	133	. .	. .	. .	. .	. .	. .	. .	. .	. .
2000	. .	1049	659	449	329	199	132	99	79	60	51	42	28	. .	. .	. .
3000	. .	. .	665	458	331	205	135	101	79	61	52	41	28	20	15	12
4000	. .	. .	. .	461	336	209	140	104	79	61	51	41	28	20	16	13
5000	. .	. .	. .	461	335	206	140	104	78	61	51	41	28	19	14	11
6000	. .	. .	. .	. .	336	205	140	103	78	61	51	41	28	19	14	11
7000	. .	. .	. .	. .	. .	204	139	103	78	60	49	41	28	19	14	11
8000	. .	. .	. .	. .	. .	204	138	102	77	59	48	40	28	19	13	11
9000	. .	. .	. .	. .	. .	204	137	100	76	58	47	39	27	18	13	10
10000	. .	. .	. .	. .	. .	201	135	99	75	57	46	38	26	18	13	10
11000	. .	. .	. .	. .	. .	. .	132	97	72	56	45	35	24	17	13	9
12000	. .	. .	. .	. .	. .	. .	130	94	68	53	43	33	22	15	12	9
13000	. .	. .	. .	. .	. .	. .	122	90	65	50	40	30	20	13	12	9
14000	. .	. .	. .	. .	. .	. .	. .	83	58	42	33	25	18	. .	. .	. .
15000	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .

ones through the dots were found by means of the reversal method. The vertical arrow-points indicate the probable position of the normal curve by steps, and the oblique arrows give the reversal one. Several series of step curves were taken for each m so that a good average curve could be constructed. It will be noticed that the step curves all lie below the others, except the one for $m = 400$.

No figure was made for the curves, which are exhibited statistically in Table IX, of December 1, 1906.

The work up to this point indicates that the thicker rods have smaller demagnetizing factors than the thin rods. To test this matter

TABLE V. [Figure 10.]

October 20, 1906.

Diam. = 0.3969 cm. = 5/32 in.

Reversals.

B.	Values of $N \times 10^4$.												
	m = 30	40	50	60	70	80	90	100	125	150	200	250	300
1000	327	199	133	95	74	59	48	38	23	22	16	. .	. .
2000	345	211	141	103	80	62	49	41	30	20	11	7	5
3000	353	216	145	107	82	64	50	42	29	20	11	7	5
4000	354	216	148	107	82	64	51	42	29	20	12	8	6
5000	357	217	147	107	83	64	52	42	29	20	12	7	6
6000	355	216	146	107	82	64	52	42	29	20	12	7	5
7000	. .	217	147	108	83	64	52	42	30	20	11	8	6
8000	. .	217	145	107	82	64	52	42	28	20	11	7	5
9000	. .	215	146	107	82	64	52	42	28	20	11	7	6
10000	. .	214	145	106	81	63	51	42	27	20	12	8	6
11000	. .	214	144	107	80	62	49	41	27	20	12	8	7
12000	. .	214	143	104	79	60	48	40	26	19	12	10	9
13000	. .	. .	141	102	76	59	46	38	24	18	13	. .	. .
14000	. .	. .	130	93	70	54	41	34	19	16	14	. .	. .
15000	. .	. .	. .	79	60	48	32	27	17	14	. .	. .	. .

more carefully, a very long solenoid was built, probably the only one of its size ever constructed. The wire was wound in a double coil over a thick brass tube, making in all eight layers. The wire used was the Annunciator No.18, of diameter = 1 mm., with red insulation. The dimensions of the solenoid are:

Length of windings = 485.3 cms. = 15 ft. 11 $\frac{3}{16}$ in.
Outside diameter = 5.96 cms.
Inside diameter = 2.86 cms.
Number of turns = 10452 for each of the two coils.

TABLE VI. [No Figure.]

November 6, 1906.

Diam. = 0.2381 cm. = 3/32 in.

Reversals.

$B.$	Values of $N \times 10^4$.						
	$m = 50$	60	80	100	150	200	300
1000	(180)	102	(54)	40	19	..	..
2000	(165)	110	63	42	20	(6)	..
3000	160	110	65	43	20	(9)	..
4000	160	113	67	43	20	12	(5)
5000	159	113	67	43	20	12	8
6000	159	114	68	43	20	12	7
7000	158	113	67	43	19	12	7
8000	158	113	66	42	19	11	7
9000	157	112	65	42	18	10	7
10000	159	112	64	41	18	9	5
11000	158	112	63	39	17	8	(3)
12000	153	108	61	36	15	7	(3)
13000	150	104	58	34	15	7	..
14000	143	97	50	29	10	5	..
15000	..	..	38	22	9	..	..

The two coils were used in parallel, so that the magnetizing field is $H' = 27.064$ c.g.s. units for each ampere.

The first rod tried in this solenoid was one of 0.9525 cm. diameter (= $\frac{3}{8}$ inch), and was a complete failure, although it gave some very interesting results. No two consecutive step method magnetization curves would agree. The rod was 15 feet long, so that $m = 480$.

The rod was carefully demagnetized and magnetized, apparently under similar conditions each time. Parts of eight different magnetization curves are shown in Figure 12 and illustrate the wide divergence at the higher inductions. The reason for this peculiar behavior of the iron was made clear when the rod was demagnetized and taken out of

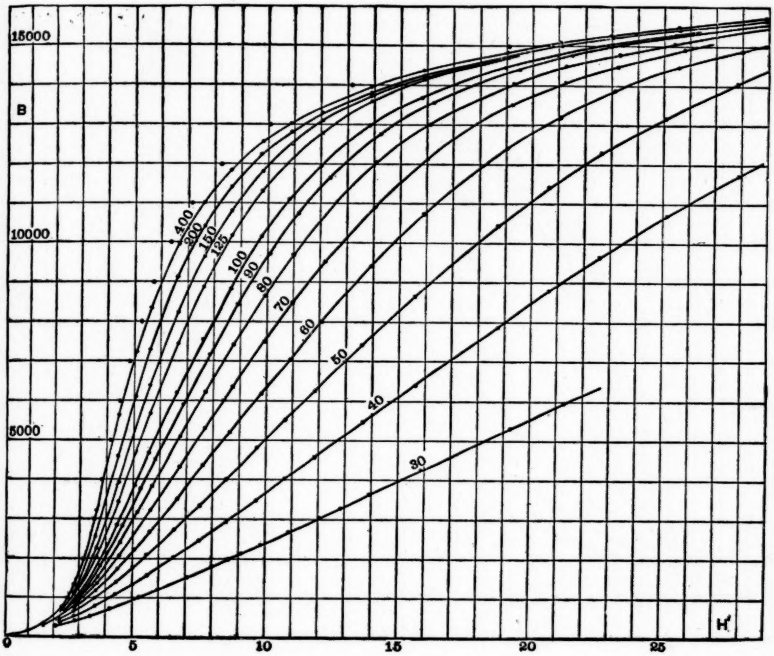

FIGURE 10. [TABLE V.]

Reversal magnetization curves for a Bessemer soft steel rod of diameter 0.3969 cm.

the solenoid, and then tested with a small pocket compass for consequent poles. It was found that the rod was quite strongly magnetized, and had polarity in the order N–S–N–S, the two middle poles being both near the middle of the rod. Evidently this rod had once been lifted around a warehouse by means of an electric crane with an electromagnet lifting device, so that it had been subjected to quite

a high magnetizing field. Besides, it is probable that the iron of this particular rod, which was not of the usual Bessemer steel, is not very homogeneous. In such cases it has been the experience of men who have had much to do with magnetization of iron in a practical way — as, for instance, Mr. Thompson, the mechanic of the Jefferson Physical

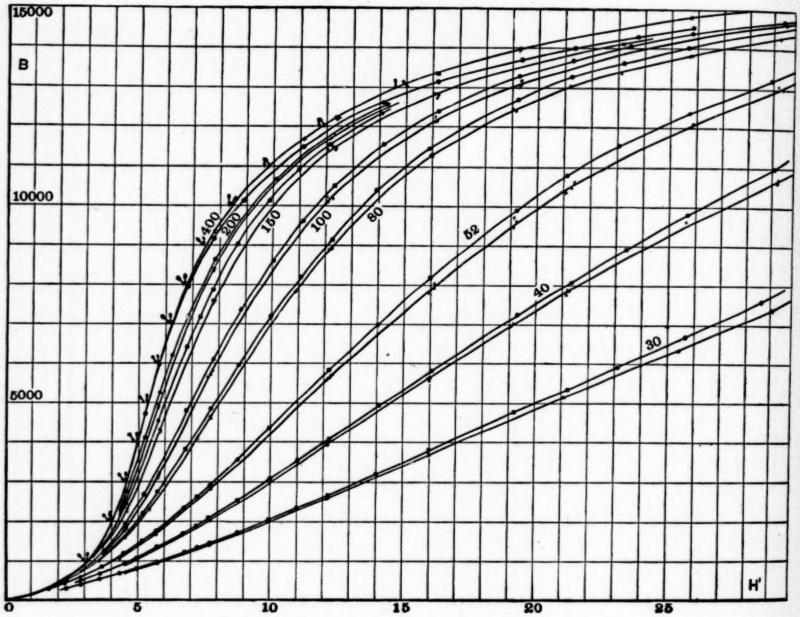

FIGURE 11. [TABLES VII AND VIII.]

Step and reversal magnetization curves for a Bessemer soft steel rod of diameter 0.3175 cm.

Laboratory — that heating the iron specimen white hot and then allowing it to cool slowly will not get rid of the consequent poles. Nor will subjecting the iron to higher magnetizing fields, and then decreasing the field while reversing constantly, so as to demagnetize, help the matter, for the poles come back straightway in their old positions.

After this the iron rods used in the long solenoid were carefully tested

for consequent poles before they were bought for the work. Even then some peculiarities were noted in the results, which are due to some irregularity in the polarity which was not apparent in the test with a small compass needle. It should be noticed that such irregularities as

TABLE VII. [Figure 11.]

November 16, 1906.

Diam. = 0.3175 cm. = 1/8 in.

Step Method.

B.	Values of $N \times 10^4$.							
	m = 30	40	52	80	100	150	200	400
1000	376	227	142	68	46	. .	. .	. .
2000	382	230	145	69	46	22	11	. .
3000	381	232	148	69	46	21	10	. .
4000	382	230	149	69	46	22	11	1
5000	386	232	148	69	46	23	12	1
6000	388	232	149	69	46	23	12	1
7000	389	234	150	69	46	23	13	1
8000	. .	234	150	69	46	23	13	1
9000	. .	237	150	69	47	22	13	. .
10000	. .	237	149	68	45	22	12	. .
11000	. .	237	147	66	43	20	12	. .
12000	. .	. .	146	65	42	20	12	. .
13000	. .	. .	142	63	40	20	11	. .
14000	. .	. .	. .	. .	. .	. .	. .	. .
15000	. .	. .	. .	. .	. .	. .	. .	. .

shown in Figure 12 are very much more pronounced when the step method is used. In fact, with the reversals it would probably turn out that a very smooth curve would be obtained, but which would lead to erroneous results in the demagnetizing factor.

No figure is given for the series whose results are tabulated in Table X, of January 16, 1907. This table should be compared with

that for the rod of same diameter worked out beginning on October 9. It will be noticed that these values for N are considerably larger than those of the earlier series. This again shows very clearly the difference between the reversal and the step method.

TABLE VIII. [Figure 11.]

November 16, 1906.

Diam. = 0.3175 cm. = 1/8 in.

Reversals.

B.	Values of $N \times 10^4$.							
	m = 30	40	52	80	100	150	200	400
1000	365	224	136	64	44	..	..	..
2000	372	227	142	65	44	19	..	..
3000	371	227	143	65	44	19	10	..
4000	372	228	145	67	44	19	11	4
5000	372	228	142	67	45	20	12	4
6000	372	227	144	67	44	20	12	4
7000	372	228	144	67	44	20	12	4
8000	368	228	143	68	44	20	12	4
9000	..	228	142	66	43	19	11	4
10000	..	226	140	63	41	17	10	4
11000	..	222	134	59	38	15	9	4
12000	..	..	131	55	34	14	8	4
13000	..	..	125	49	30	..	..	..
14000	..	..	..	42	(22)	..	..	..
15000	..	..	..	..	..	..	..	..

Figure 13 gives the experimental curves corresponding to Table XI, January 18, 1907. They were taken by the step method, and each curve was based on three or four separate magnetizations from zero to the highest value of H', so that good average results might be obtained. It will be noticed that the curve for m = 200 passes very nearly through two sets of observations, but that on either side of it lie

observation-points at quite a distance off. Most of the other curves are in much better agreement with their points. There were also taken a number of magnetization curves for the initial length of the rod, 15 feet, which made $\mathfrak{m} = 329$; these curves resembled the ones

TABLE IX. [No Figure.]

December 1, 1906.

Diam. = 0.6350 cm. = 1/4 in.

Reversals.

B.	Values of $N \times 10^4$.			
	$\mathfrak{m} = 50.$	60	80	100
1000	. .	. .	. .	. .
2000	137	107	64	39
3000	144	105	61	39
4000	143	105	60	38
5000	145	105	60	40
6000	145	105	60	38
7000	144	103	61	39
8000	141	102	59	38
9000	141	101	58	37
10000	141	99	56	37
11000	142	98	55	36
12000	140	96	52	34
13000	136	93	48	33
14000	. .	87	47	32
15000	. .	. .	. .	. .

for the rod with pronounced consequent poles. It thus appears that there must have been some irregularity in the demagnetized rod near one or perhaps both ends of the rod. As the rod was cut down from $\mathfrak{m} = 329$ to $\mathfrak{m} = 200$, most of these irregularities were cut off. Then at the next shortening practically all the rest was eliminated. For $\mathfrak{m} = 30$ a reversal curve, represented in the figure by crosses, was also taken.

See Figure 14 for the original curves, from $\mathfrak{m} = 15$ to $\mathfrak{m} = 240$, from which Table XII, of January 22, 1907, was constructed. It will be seen that on the figure there appear a number of crosses. These represent magnetization curves, not actually drawn, which were taken with the

TABLE X. [No Figure.]

January 15, 1907.

Diam. = 0.4763 cm. = 3/16 in.

Step Method. Long Coil.

B.	Values of $N \times 10^4$.				
	$\mathfrak{m}$ = 80	100	150	200	300
1000	66	40	18	. .	. .
2000	66	43	19	11	4
3000	65	43	20	11	4
4000	66	43	20	11	4
5000	66	43	21	12	4
6000	66	42	22	12	4.5
7000	66	42	21	11	4.5
8000	65	42	20	10.5	4
9000	65	42	19	10.5	4
10000	64	41	18	10.5	4
11000	62	40	18	10.5	4
12000	59	38	16	10	4
13000	54	33	15	9	3
14000	47	28	14	8	2
15000	37	25	. .	. .	2

reversal method. This brings out a most interesting point. The thick brass tube opposes a sudden change in the magnetizing field, by virtue of eddy currents, and thus the establishment of the field is somewhat delayed and the magnetization of the iron takes place more slowly. The step method magnetization also is slower than the step method when used in a plain solenoid wound on a tube of pasteboard, as is the

first solenoid. But as the reversal method has now almost overtaken the step method, we may conclude that both are very nearly at their limiting positions, reached for very slow establishment of the magnetizing field, which are probably very nearly the same.

TABLE XI. [FIGURE 13.]

January 18, 1907.

Diam. = 1.111 cms. = 7/16 in.

STEP METHOD. LONG COIL.

B.	Values of $N \times 10^4$.							
	m = 30	40	50	60	80	100	150	200
1000	341	202	141	98	66	39	. .	. .
2000	347	208	144	103	66	41	20	11
3000	348	208	145	105	67	41	20	11
4000	348	207	146	106	66	41	21	11
5000	348	210	144	106	65	41	21	11
6000	351	211	145	106	66	41	22	11
7000	351	213	145	107	66	41	21	10
8000	351	214	145	107	66	41	21	10
9000	351	213	145	106	65	41	21	9
10000	. .	210	144	104	63	40	20	8
11000	. .	211	142	103	62	40	20	8
12000	. .	. .	140	100	60	38	20	7
13000	. .	. .	140	98	60	36	19	6
14000	. .	. .	. .	93	59	35	19	4
15000	. .	. .	. .	. .	51	34	17	3

Figure 15 gives the original curves of Table XIII, taken on February 21, 1907, and following. As it was found that in the long solenoid the reversal method gives us practically the same results as the steps method, it was now used throughout because of its convenience and accuracy. Compared with the results of the rod of "cold rolled shafting" these values are somewhat smaller, but not more perhaps than is

due to the slight difference between the step and reversal methods which still remains. It is thus probable that the material of these two rods is not of very great importance. The curve for $m = 240$ was also taken, but was very nearly coincident with that for $m = 200$.

When this rod, which we will call Rod No. I, was tested for consequent poles, there was also selected another one of the same diameter

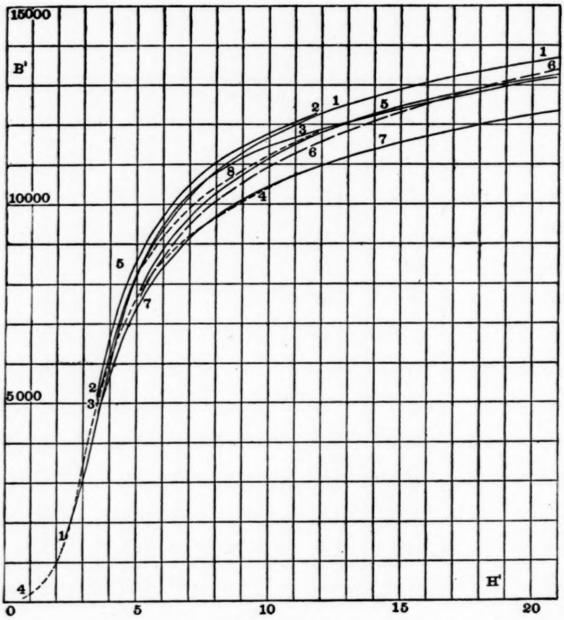

FIGURE 12.

Effect of consequent poles in an iron rod. The magnetization curves shown were taken under apparently the same conditions.

from the same lot of iron. Both were 20 feet long, and pieces of 1 foot and 4 feet were cut off from the ends. Rod No. II was magnetized at $m = 240$, and gave the higher curve marked by the crosses. The pieces of 4 feet length had been mixed up so that it was impossible to say which belonged to Rod No. I and which to the other one. Test pieces of $m = 60$ were now prepared from both of these pieces, all of these rods of diameter 1.905 cms. being wound with 50 secondary turns

in the centre. The short rods now gave the magnetization curves which are merely indicated by crosses near the curves for $\mathfrak{m} = 80$ and $\mathfrak{m} = 60$ of Rod No. I. It is now evident which rod each of the small pieces came from. Of course the magnetic induction was now measured at a distance

TABLE XII. [Figure 14.]

January 22, 1907.

Diam. = 1.905 cms. = 3/4 in. Cold Rolled Shafting.

Step Method. Long Coil.

B.	Values of $N \times 10^4$.											
	$\mathfrak{m} = 10$	15	20	30	40	50	60	80	100	150	200	240
1000	1960	1067	661	338	195	140	99	61	..	..	..	..
2000	1954	1064	663	333	198	147	100	63	40	23	..	..
3000	..	1075	673	342	203	150	107	63	41	21	(6)	..
4000	..	..	671	344	207	150	107	63	41	21	8	1
5000	..	..	669	344	208	148	106	63	41	21	9	2
6000	..	..	..	341	210	148	103	61	39	21	10	3
7000	..	..	..	342	210	146	102	60	38	21	12	5
8000	..	..	..	338	208	144	100	58	37	21	13	5
9000	..	..	..	341	207	141	98	58	36	19	13	5
10000	..	..	..	..	204	137	96	56	34	19	12	5
11000	..	..	..	..	200	134	93	54	32	19	12	5
12000	..	..	..	..	..	129	87	51	29	18	12	5
13000	..	..	..	..	..	124	81	47	25	18	12	..
14000	..	..	..	..	..	..	76	45	23	..	9	..
15000	..	..	..	..	..	..	..	..	..	..	..	..

of about 9.5 feet in the original 20 feet rods, but still the normal curves would probably not differ much. On the other hand, the normal curve for Rod No. I is quite different from that for Rod No. II.

With the help of the tracing cloth scale to be described below, Figure 16 was constructed, it being assumed that the maximum I is practi-

cally reached when $B = 17,000$. This body of N-curves shows the curvatures which we were led to expect, and also the tremendous turn to the left as the curves get near the point of complete saturation. This curve might be said to embody the most important results obtained about the N-curves. The one corresponding to $\mathfrak{m} = 20$, after going

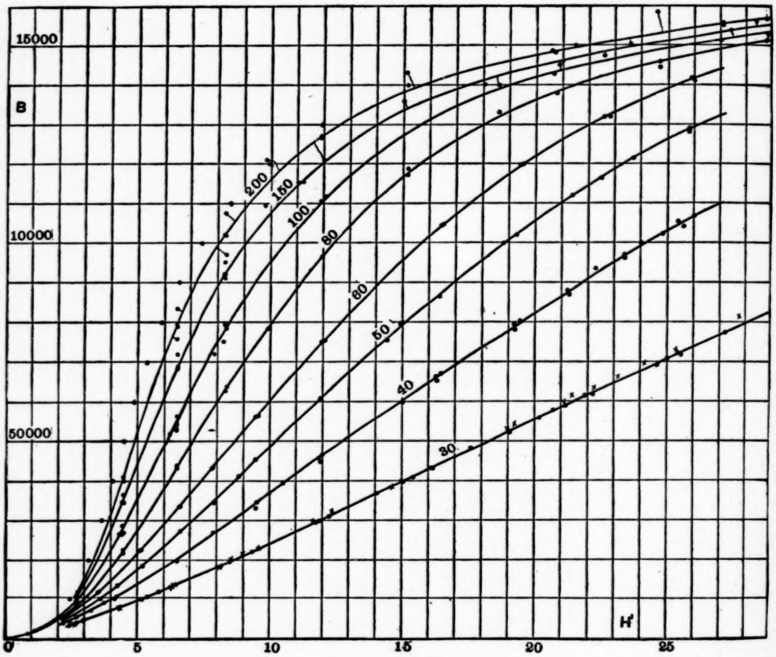

FIGURE 13. [TABLE XI.]

Step magnetization curves in long coil for a Bessemer soft steel rod of diameter 1.111 cms.

out nearly straight far beyond the limits of the figure, sweeps back to the left and just shows in the upper left-hand corner. It will be noticed that the points of observation for all the curves become uncertain after $B = 12,000$; this is to be expected because the magnetization curves there become almost horizontal and run into one another, and the finding of the abscissa-differences is a very difficult matter.

METHOD OF REDUCING OBSERVATIONS.

As a typical illustration of the whole work, let us consider the reduction of the observations taken on the largest iron rod used in the long

TABLE XIII. [FIGURE 15.]

February 21, 1907.

Diam. = 1.905 cms. = 3/4 in. Bessemer Steel.

REVERSALS IN LONG COIL.

B.	Values of $N \times 10^4$.									
	m = 15	20	30	40	50	60	80	100	150	200
1000	1009	658	332	201	139	98	64	39	20	9
2000	1019	663	331	211	141	102	61	41	20	10
3000	1032	668	336	209	140	102	62	41	21	10
4000	1032	665	339	212	144	102	62	41	19	11
5000	1042	657	340	213	142	103	63	42	20	11
6000	1045	659	335	207	140	103	62	40	20	10
7000	1040	662	335	207	141	102	61	40	20	11
8000	. .	662	335	204	138	99	58	38	21	12
9000	. .	661	332	200	136	97	55	39	19	12
10000	. .	662	327	197	131	95	52	34	18	11
11000	. .	. .	324	194	128	90	51	31	17	11
12000	. .	. .	320	188	123	84	46	30	15	10
13000	. .	. .	315	185	117	79	39	27	14	9
14000	. .	. .	303	171	104	73	36	20	14	7
15000	. .	. .	. .	158	92	71	28	. .	13	7

solenoid. This is the series on Rod No. I, begun on February 21. It usually takes about two days to take a series of observations, and the reductions and plotting of curves take about two or three days more.

When using the reversal method, the observations were taken under

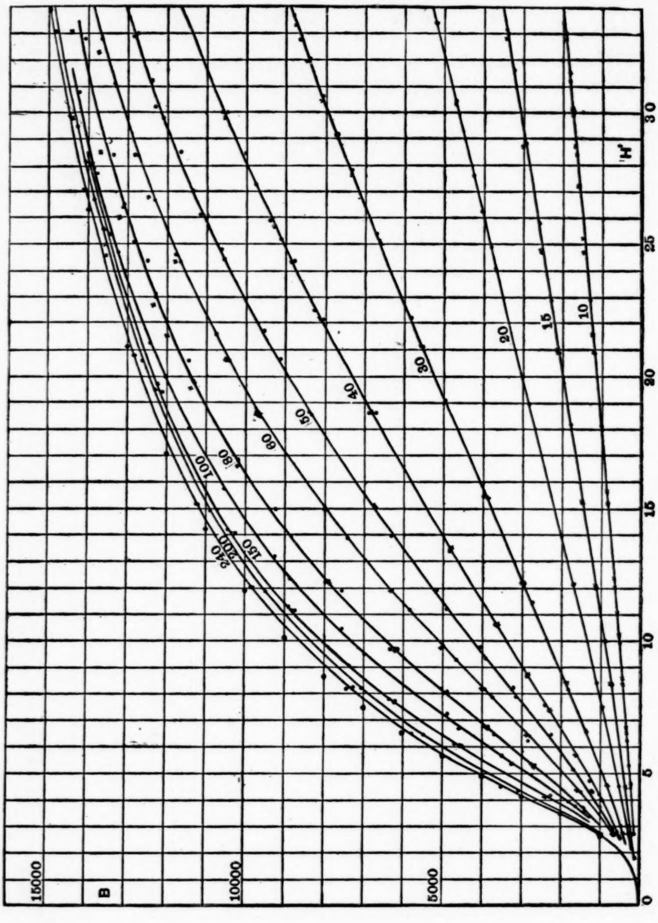

FIGURE 14. [TABLE XII.]

Step magnetization curves in long coil for a rod of "cold rolled shafting" of diameter 1.905 cms.

the headings: current in solenoid, resistance in the box R', and ballistic throws observed. In the case of the step-by-step method the zero reading of the galvanometer was also necessary.

We start from the fundamental equation of a current through whose circuit the magnetic flux varies:

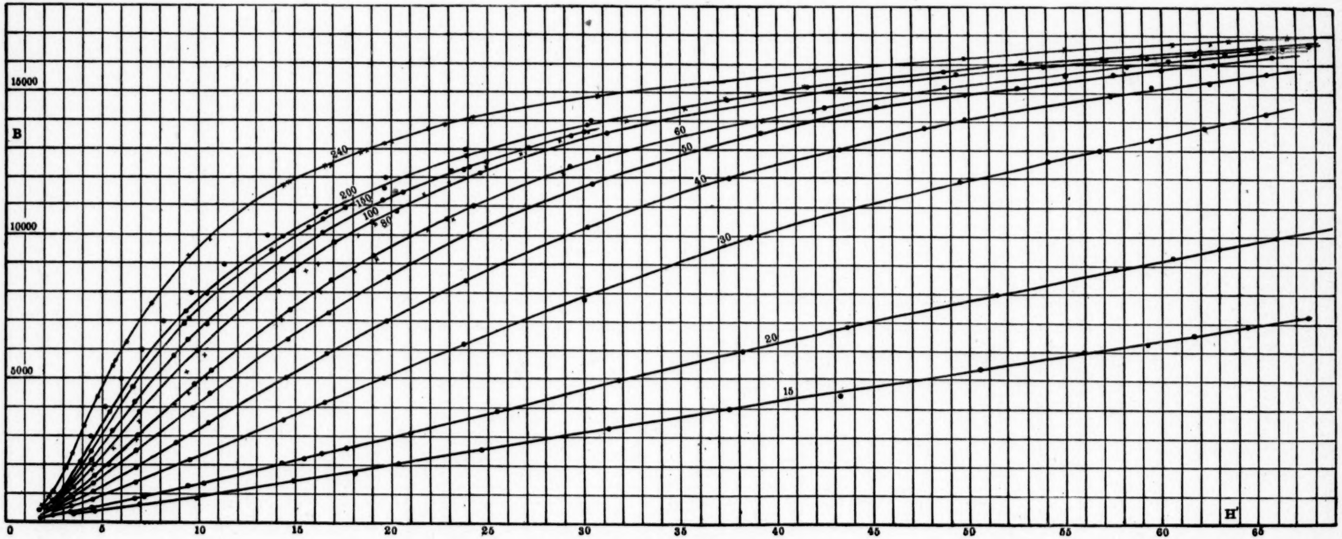

FIGURE 15. [TABLE XIII.]

Reversal magnetization curves in long coil for rods of Bessemer soft steel of diameter 1.905 cms.

$$E - \frac{dN}{dt} = CR,$$

where $E =$ electromotive force in the circuit, not due to changes in flux,

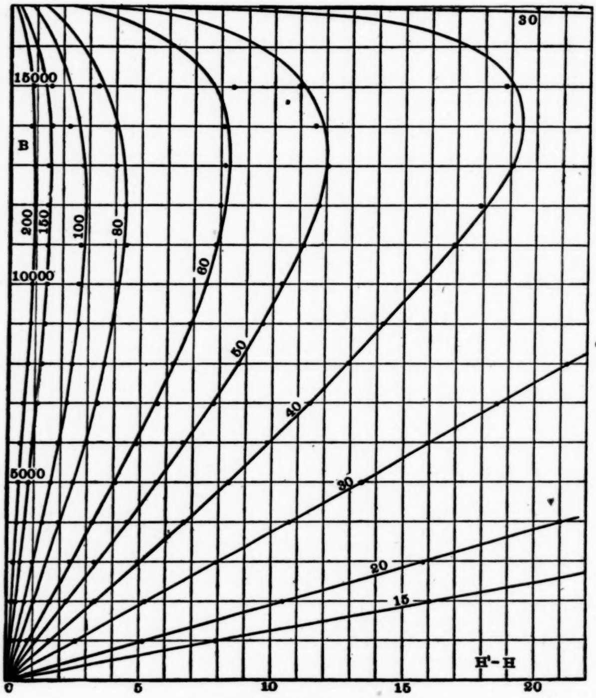

FIGURE 16. [TABLE XIII.]

Back-shearing curves for Bessemer soft steel rod of diameter 1.905 cms.

$N =$ total magnetic flux of induction through the circuit in the direction of the magnetic lines due to the current C, times the number of turns of wire in the circuit,

$t =$ time variable,

$C =$ actual current at time t flowing in the direction in which E acts,

R = total resistance of the circuit.

If we apply this equation to our secondary coil circuit, which includes the ballistic galvanometer, we have, since $E = 0$,

$$\Delta N = R \int_0^t C \cdot dt = RQ,$$

or

$$Q = \Delta N / R,$$

where Q = total charge through galvanometer,

ΔN = number of flux-turns of change in the magnetic induction through the circuit.

This equation is expressed in c.g.s. units. If we use as our units the ampere, ohm, microcoulomb, and gauss, as we have done, then we must use the equation,

$$Q = \Delta N / (100 R).$$

We have also

$$Q = T/S,$$

where T = actual throw in centimeters of scale reading produced by the discharge of Q microcoulombs through the galvanometer, and S = sensitiveness of galvanometer, expressed in centimeters of deflection obtained by discharging 1 microcoulomb through the galvanometer.

Now in the reversal method as used in these experiments,

$$\Delta N = 2\,BAn = 2\,B\pi(D/2)^2 n,$$

where B = the magnetic induction in gausses, or number of lines of induction per square centimeter passing through the middle of the iron rod,

A = cross-section of rod in square centimeters,

n = number of turns of secondary coil wound around the middle of the rod,

D = diameter of the rod, as before.

This gives us

$$\frac{2\,B\pi(D/2)^2 n}{100 \cdot R} = \frac{T}{S},$$

or

$$\frac{B}{T} = \frac{100 \cdot R}{2\,S\pi(D/2)^2 \cdot n}.$$

This formula is the most convenient for our purposes. As in our series we had the data

$$S = 1.489$$
$$D = 1.905 \text{ cms.}$$
$$n = 50 \text{ turns}$$

we get
$$\frac{B}{T} = \frac{100 \cdot R}{2(1.489)\,\pi\,(0.9525)^2 \cdot 50}.$$

The right-hand member is a constant for any given R. In the work on the series of curves the R had values ranging from 117 to 7117 ohms; the galvanometer and secondary coil circuit having itself 117 ohms, of which the galvanometer had about 99 ohms, and the coil 18 ohms, the other resistance being added, when convenient, from the resistance box R'. The constants for these various R's were found and written down. Then all we have to do to find the B for any observation is to multiply the observed throw in centimeters by the proper constant. This was done either by means of logarithms or a very good slide rule.

If we use the step-by-step method, the formula simply drops the factor 2 and becomes,

$$\frac{\Delta B}{T} = \frac{100\,R}{S\pi(D/2)^2 n}.$$

For the long solenoid we have simply

$$H' = \frac{4\,\pi N}{10\,L} \text{ (No. of amperes used)}$$
$$= 27.064 \text{ (No. of amperes).}$$

Having found the values of B and H', they were multiplied by 3 and 2 respectively, in order to facilitate the plotting of the points of observation. Then the magnetization curves were drawn by free-hand so as to fit the points as closely as possible.

This gives us the curves from $\mathfrak{m} = 15$ to 200 in Figure 15. To find the corresponding normal curve ($\mathfrak{m} = \infty$) a graphical device was found to be of the very greatest utility. Not only was an enormous amount of time saved, which otherwise it would have been necessary to spend in almost endless computations, but the device was a positive aid in determining the position of the normal curve. On a large sheet of tracing cloth were drawn about seventeen horizontal lines, so that when properly placed over the sheet of millimeter paper on which the magnetization curves had been drawn, they coincided with the lines $B = 0$, 1000, 2000, etc., up to 16,000. By means of lines radiating out from a point on the lowest of these horizontal lines, each one of the lines

above was divided into a large number of equal intercepts, each of which represented exactly 0.0010 of N, the demagnetizing factor, for the particular B corresponding to the line. The larger of these intercepts were further subdivided into tenths by means of short dashes, and each horizontal line was numbered for every 0.0010, beginning from zero on the left. Thus the tracing cloth was simply a large transparent scale through which the N corresponding to every H_i could be immediately read off. The error in the inaccurate spacing of the divisions of the scale was about 1 part in 200.

Now suppose we arbitrarily say for the moment that the N for the curve $\mathfrak{m} = 200$, all along the curve, shall be 0.0016, or the value of N for the corresponding ellipsoid of revolution. By placing the tracing cloth so that any desired line coincides with its corresponding B below, and the magnetization curve for $\mathfrak{m} = 200$ crosses at $N = 16$ units, we can read off the number of units for each of the other curves. After doing this for all of the horizontal lines of our scale, we have a table of values similar to that given for the rod of February 21, only the column for $\mathfrak{m} = 200$ will consist wholly of numbers 16.

This table is thus our first approximation. We may now put away our magnetization curve sheet with the scale, and proceed to get a better approximation by merely studying the table. It will be noticed that all the other columns will have values less than for the corresponding ellipsoids. The only logical thing to do is to decrease the 16's somewhat, at the same time decreasing every other number in the same row by the same amount, so as to give a table consistent as a whole when compared with the table for ellipsoids; and this gives us something similar to the table given. At the best approximation, the values for $\mathfrak{m} = 200$ will still be a unit or two in doubt, but this will make but a small error in the rods 30 to 50 diameters long. Of course individual values of N in the table are subject to errors in the drawing of the curve as well as observational errors, but when all the values of N for a certain length of rod are considered, a smooth curve could easily be drawn throughout the range of B in the experiment. We have, however, preferred to leave the tables as given directly from the last approximation.

Should any one not be quite satisfied with the values as tabulated for any one series of experiments, he may easily change the whole table to suit himself, but he must do this subject to the condition of adding or subtracting the same number for any one row as it is given here.

TABLE XIV.

Observer.	Method.	D.	L.	Length Solenoid.	H'.	Range in m used.	Remarks.
Ewing, 1885	Ball. Steps	0.158	47.5 to 7.9	. .	0–35	300–50	
Tanaka-daté, 1888	Magn.	.100	9	9.25	. .	90	Made in Japan.
	Gauss A	.153	2–6	11.9	. .	13.1–39.2	Made in England.
	Ewing's	.115	33.4	38.4	. .	. .	" "
C. R. Mann, 1895	Magn. Gauss A	2.370 –.237	11.850	30	20–1300	5–50	L constant, D turned down.
	"	1.924 –.1924	9.620	30	22–660	5–50	L constant, D turned down.
	"	0.0836	25.08 –4.18	38.5	2–300	300–50	D constant, L cut down.
Benedicks, 1902	Magn.	0.8	20	. .	23–206	25	All observations made on hysteresis cycles. Normal curve obtained by ellipsoid results.
	Ball. Steps	0.8	20	. .	23–206	25	
Jefferson Physical Laboratory, 1907	Ball. Rev.'s	0.2381	182.8 –11.91	207.7	1–26.3	768–50	397 sec. turns.
	"	0.3175	182 8 –9.53	"	1–30	576–30	230 " "
	Steps	"	"	"	"	"	" " "
	Rev.'s	0.3969	182.8 –11.91	"	1–29	461–30	180 " "
	"	0.4763	182.8 –4.76	"	3.4 –22.3	384–10	130 " "
	"	0.6350	182.8 –6.35	"	3.7 –22	288–10	100 " "
	"	"	63.5 –31.75	"	4.7 –30.8	100–50	30 " "
	"	1.270	182.8 –12.70	"	3.4 –20.3	144–10	60 " "
	Steps	0.4763	182.8 –38.10	485.3	1.8–26	384–80	195 " "
	"	1.111	366.4 –33.33	"	1–34	329.5 –30	50 " "
	Stepsand Rev.'s	1.905	457.2 –19.05	"	2.4–33.7	240–10	50 " "
	Rev.'s	1.905	457.2 –28.58	"	1.8–44	240–15	50 " "

Discussion of Investigations on the Demagnetizing Factors.

It was considered worth while to collate briefly the leading experimental conditions which have been used in the determinations of N for iron cylinders. Table XIV on the preceding page has therefore been constructed from available data.

It will be noticed that Mann used some very thick iron bars in the first two of his experimental series. However, a given diameter remained constant only throughout a single magnetization curve, say for $\mathfrak{m} = 5$; after this the bar was turned down to a smaller diameter on the lathe, so that $\mathfrak{m}$ was thereby increased. If now the ballistically

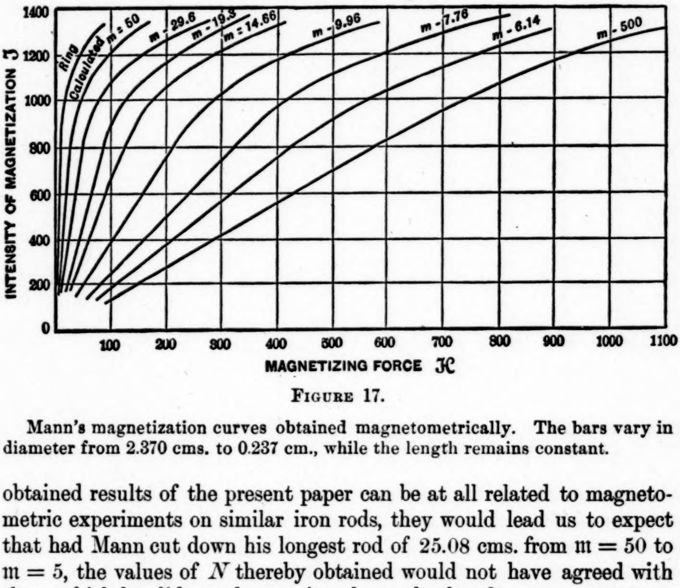

FIGURE 17.

Mann's magnetization curves obtained magnetometrically. The bars vary in diameter from 2.370 cms. to 0.237 cm., while the length remains constant.

obtained results of the present paper can be at all related to magnetometric experiments on similar iron rods, they would lead us to expect that had Mann cut down his longest rod of 25.08 cms. from $\mathfrak{m} = 50$ to $\mathfrak{m} = 5$, the values of N thereby obtained would not have agreed with those which he did get by turning down the bar from $\mathfrak{m} = 5$ to $\mathfrak{m} = 50$. In fact the two sets of values for N, belonging to the two methods "sawing off" and "turning down" respectively, would probably have diverged more and more as $\mathfrak{m}$ was decreased, the "turning down" values for N being always less because the diameters of the bars of this method are the greater, as carried out.

As noted in the outline at the head of this paper, Mann found that

the values of N as determined magnetometrically are nearly constant up to $I = 800$, but after this they increase enormously. This behavior

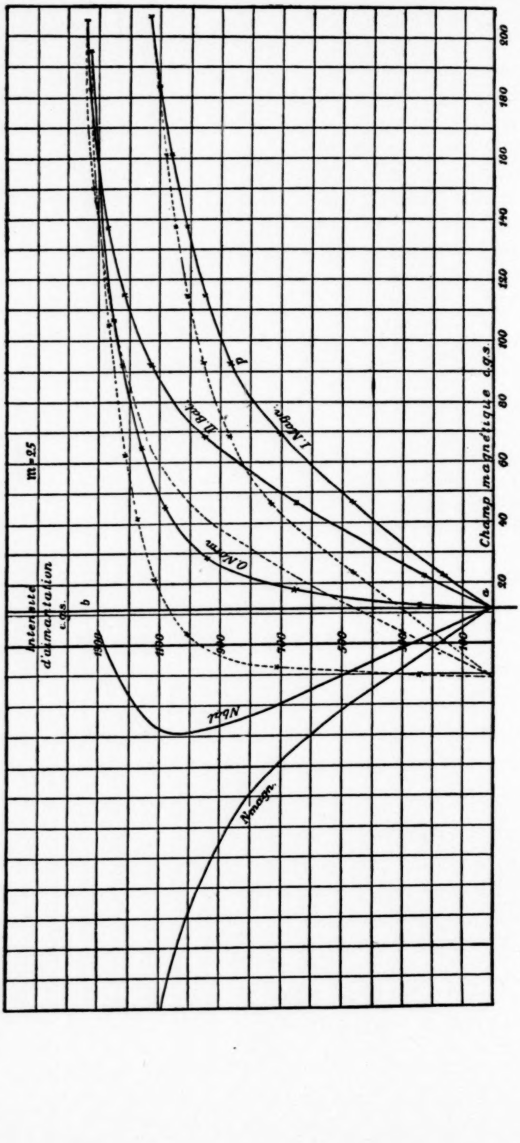

FIGURE 18.

Benedicks's magnetization curves.

of the N-curves is undoubtedly closely related to the change in the pole-distance ratio l/L, which probably approaches the value unity for complete saturation. The magnetization curves taken magnetometrically tend to diverge, or spread apart, for high magnetizations, whereas those taken ballistically all converge rapidly to the maximum ordinate I_{∞}. Figure 17 is reproduced from Mann's paper,[16] and shows the curves from $\mathfrak{m} = 5$ to $\mathfrak{m} = 50$ obtained from his first cylinder. The method by which Mann gets at the position of the "normal" magnetization curve for an infinite rod is to assume that the magnetometric N for a cylindrical rod of $\mathfrak{m} = 300$ is the same as for an ellipsoid of the same length and central cross-section, namely $N = 0.00075$.

In his investigation Benedicks obtained the value of N for only one rod of hard steel ($\mathfrak{m} = 25$), but did this very thoroughly, using both the ballistic step and magnetometric methods. His normal curve is determined by transforming the steel cylinder into an ellipsoid of $\mathfrak{m} = 30$, obtaining magnetometrically the magnetization curve for this ellipsoid, and back-shearing this curve into the normal curve by means of the known demagnetizing factor for this ellipsoid, which is $N = 0.0432$. Theoretically the method is perfect, but we rather doubt whether it can be depended upon to give uniformly agreeing results in practice. The magnetization curves obtained by Benedicks are shown in Figure 18, which has been reproduced from his article[17]. The figure shows the two types of N-curves, — the magnetometric and the ballistic, — and their opposite behavior for high magnetizations. Benedicks also publishes the N-curves as he derives them from Ewing's original six curves, all showing a behavior similar to that of his own curve N_{bal}. These N-curves are practically identical with those shown in Figure 19 of this paper; these were determined by our methods directly from Ewing's curves shown in Figure 2, which were reconstructed from the original figure[18] in order to have both figures on exactly the same scale as our own curves, for purposes of comparison. See Figure 16, which shows the N-curves for our Bessemer steel rod of diameter 1.905 cms.

We might note that Benedicks gets no curvature in the N-curve near the origin, because he takes his observations from hysteresis cycles of magnetization, the maximum applied field being about $H' = 206$ units.

Benedicks criticizes Mann's assumption that $N = 0.00075$ for an

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[16] Phys. Rev., **3**, 359–369 (1896).
[17] Bihang Svenska, Vet.-Akad. Handlingar, **27** (1), No. 4, 14 pages (1902).
[18] Phil. Trans., **176** (1885), Plate 57, Figure 3.

iron cylinder of $m = 300$, as being unwarranted. He determines N by both the ballistic and magnetometric methods for a rod of $m = 300$ by back-shearing the ballistic curve into the normal curve, using $N_{bal} = 0.0005$, according to Du Bois, thus finding the N to be 0.0028 for the magnetometric method. He would, therefore, correct Mann's

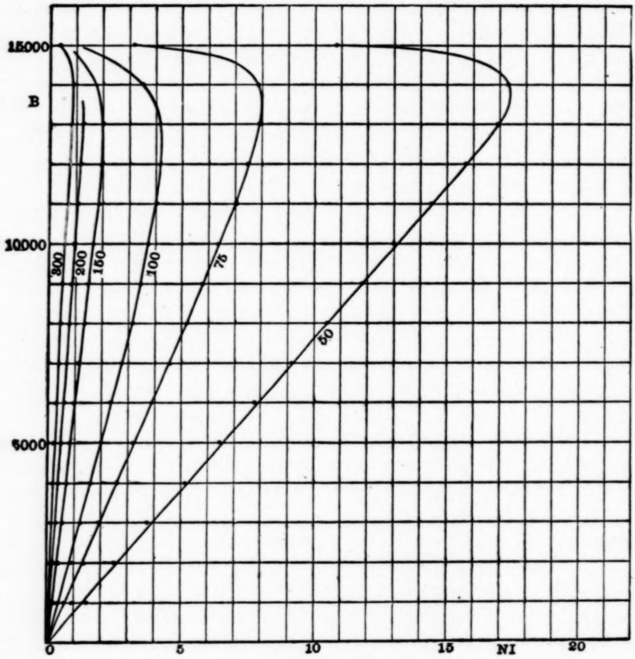

FIGURE 19.

Back-shearing curves for Ewing's soft iron wire of diameter 0.158 cm. Determined from results found in the present paper.

values of N by adding 0.0020 to each N throughout. Now it seems to us quite clear, as remarked somewhere in the earlier part of this paper, that we have no right to assume that the normal I vs. H curve, as obtained ballistically, should be even approximately the same as the Mean I vs. Mean H curve of the magnetometric method. This assumption is rendered particularly doubtful when we see the very wide

difference between the magnetization curves for m = 300 by the ballistic and magnetometric methods as observed by Benedicks and published in the "Bihang," and when we consider at the same time that both these curves cannot possibly be very far away from their

TABLE XV.

VALUES OF N.

m.	Ellipsoid.	Cylinder.				
		Ballistic.			Magnetometric.	
		Du Bois.	Benedicks.	Jeff. Phys. Lab.	Mann.	Benedicks.
5	0.7015	. . .	. . .	. . . . .	0.68000	. . .
10	0.2549	0.2160	. . .	0.1820–0.2001	0.25500	. . .
15	0.1350	0.1206	. . .	0.1000–0.1075	0.14000	. . .
20	0.0848	0.0775	. . .	0.0635–0.0671	0.08975	. . .
25	0.0579	0.0533	0.0444	0.0445–0.0465	0.06278	0.0658
30	0.0432	0.0393	. . .	0.0331–0.0388	0.04604	. . .
40	0.0266	0.0238	. . .	0.0204–0.0234	0.02744	. . .
50	0.0181	0.0162	. . .	0.0139–0.0160	0.01825	. . .
60	0.0132	0.0118	. . .	0.0100–0.0116	0.01311	. . .
70	0.0101	0.0089	. . .	0.0076–0.0088	0.00988	. . .
80	0.0080	0.0069	. . .	0.0060–0.0069	0.00776	. . .
90	0.0065	0.0055	. . .	0.0050–0.0056	0.00628	. . .
100	0.0054	0.0045	. . .	0.0041–0.0046	0.00518	. . .
125	0.0036	. . .	. . .	0.0028–0.0032	. . .	. . .
150	0.0026	0.0020	. . .	0.0019–0.0023	0.00251	. . .
200	0.0016	0.0011	. . .	0.0011–0.00125	0.00152	. . .
300	0.00075	0.0005	. . .	0.0004–0.0007	0.00075	. . .

limiting positions for the infinite rod. On the other hand it is quite reasonable to suppose that the N for any iron ellipsoid is always greater than the N for the corresponding cylinder, obtained by either of the two methods; because by adding the extra mass of iron to an

TABLE XVI.

The Demagnetizing Factors in the Range of Practical Constancy.

Reversals in Short Coil:

m.	$D = 0.2381.$	0.3175.	0.3969.	0.4763.	0.6350.	1.270.
10	. . .	. . .	. . .	2001	1990	1820
15	. . .	. . .	. . .	1049	1028	(1000)
20	. . .	. . .	. . .	665	653	635
25	. . .	. . .	. . .	461	(458)	(445)
30	. . .	372	355	336	332	331
40	. . .	228	216	206	205	204
50	159	(155)	147	140	139	139
60	113	(113)	107	103	101	100
70	. . .	. . .	(81)	78	76	76
80	67	67	64	61	60	62
90	(54)	(54)	52	51	(50)	50
100	43	44	42	41	41	41
125	. . .	. . .	29	28	28	(28)
150	20	20	20	19+	19	. . .
200	12	12	12	11+	. . .	. . .
300	7	. . .	6	. . .	. . .	. . .

TABLE XVII.

Principle of Step Method:

m.	Du Bois. $D = 0.158.$	$D = 0.3175.$	0.4763.	1.111.	1.905.	1.905 (Rev.'s in Long Coil).	Percentage Difference between 0.3175 and 1.905.
10	2160	. . .	. . .	. . .	1960	. . .	. . .
15	1206	. . .	. . .	. . .	1075	1045	. . .
20	775	. . .	. . .	. . .	671	662	. . .
25	533	. . .	. . .	. . .	(465)	(455)	. . .
30	393	388	. . .	350	343	336	15.5 %
40	238	234	. . .	212	209	209	12 "
50	162	(160)	. . .	145	149	142	11 "
60	118	(116)	. . .	106	106	103	11 "
70	89	(88)	. . .	. . .	. . .	. . .	. . .
80	69	69	66	66	63	62	10 "
90	55	(56)	. . .	. . .	. . .	. . .	. . .
100	45	46	43	41	41	41	12.2 "
125	. . .	. . .	. . .	. . .	. . .	. . .	. . .
150	20	23	21	21	21	20	15 "
200	11	12.5	12–	11	11	11	14 "
300	5	. . .	4	. . .	. . .	. . .	. . .

The figures in parentheses are interpolated; all others have been obtained experimentally. For purposes of comparison, the values of Du Bois are given in Table XVII. The numbers given in these tables represent $N \cdot 10^4$, as in the earlier tables.

ellipsoid in order to form the corresponding cylinder, the surface magnetism σ is shifted nearer to the ends of the rod and should exert less demagnetizing force. To be sure, we now have some volume magnetism, $\rho = -$ Divergence I, in the cylinder, which does not exist in the ellipsoid, but the effect of this is probably always extremely small. On the whole we feel certain that Mann's value is quite near the truth, and is probably even a trifle too large.

Table XV, on page 239, gives briefly all the results obtained on demagnetizing factors for the region in which they are practically constant, that is, for the iron cylinders up to about $I = 800$, or $B = 10{,}000$.

The values of N as obtained for the various diameters of rods in the present investigation are given in Tables XVI and XVII on the preceding page. They were taken from the tables given for each separate rod, and are fairly constant over the range from $B = 3000$ to $B = 9000$.

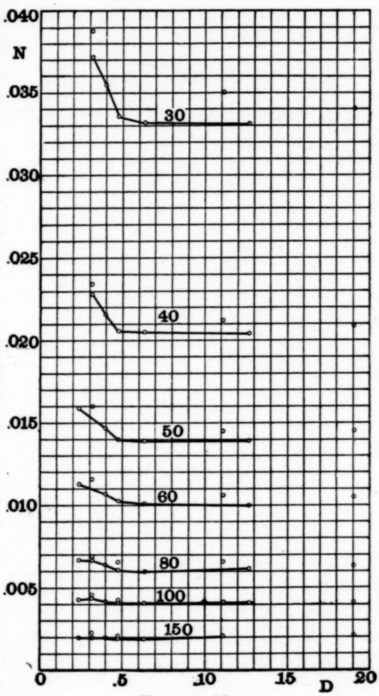

FIGURE 20.

Curves showing the variation in N for different diameters of iron rods. The numbers near curves give the corresponding values of m.

The values of N of these tables have been plotted in Figure 20 against the corresponding diameters of the rods. The points connected by straight lines are the reversal method values, while those left unconnected are the ones taken by the principle of steps. It seems to be shown that the values of N experience a rapid drop from $D = 0.238$ to about $D = 0.50$, and then remain nearly constant as the diameter is further increased.

For practical use in finding permeabilities Table XVIII has been

constructed. The induction is assumed to be observed experimentally by the step method, and the K of the table is used in the equation

$$H = H' - KB.$$

TABLE XVIII.

m	Values of K.	
	$D = 0.3175$	$D = 1.1$ to 2.0 cms.
15	. . . .	0.00852
20	. . . .	0.00533
25	. . . .	0.00366
30	0.00309	0.00273
40	0.00186	0.00166
50	0.00127	0.00116
60	0.000925	0.000845
80	0.00055	0.000505
100	0.000366	0.000326
150	0.000183	0.000167

Problem.

Suppose the magnetic susceptibility in a soft iron rod similar to Bessemer steel is to be tested ballistically. Suppose the rod is neither very thick nor long, and the ballistic galvanometer (Thomson) is not very sensitive. In order to get the greatest possible throw we may wind a large number of turns of wire of secondary coil around the middle of the rod, being careful not to exceed the point of maximum sensitiveness. This is reached when an additional turn of wire adds proportionately more resistance to that already in the galvanometer circuit than it adds turns to the total number of turns. Of course as long as the secondary coil is wound on in a single layer, and the resistance of the galvanometer is not negligible, this condition can never be reached; but where the coil is built up in several layers the resistance finally predominates. Suppose we have:

Galvanometer resistance = 12 ohms.

Sensitiveness = 0.0695 mm. throw per microcoulomb.

Dimensions of Iron Rod: Diameter = 5 mms. Length = 20 cms., so that $m = 40$.

Secondary Coil: 480 turns of fine wire. Length = 3 cms. Resistance = 19.42 ohms.

We therefore neglect the leakage of induction through the secondary coil. If we have no extra resistance in the galvanometer circuit the formula gives for the method of reversals:

$$\frac{B}{T} = \frac{100 \cdot R}{S \cdot 2\pi(0.25)^2 480} = \frac{42 \cdot 31}{0.00695\,\pi \cdot 0.60} = 2400.$$

This shows that we need no extra resistance for the secondary circuit.

Suppose we magnetize in a solenoid 31 cms. long and wound with 5 layers of wire, 113 turns in each layer. Then we have

$$H' = \frac{4\pi 565}{10 \cdot 31} \cdot (\text{No. of amperes}) = 22.9\ (\text{amperes}).$$

We get the following observations:

Current in Solenoid.	Ballistic Throw.
0.498 ampere	1.82 centimeters.
0.664 "	2.59 "
0.837 "	3.36 "
0.975 "	3.97 "
1.120 "	4.55 "
1.257 "	5.02 "

giving the calculated results:

H'.	B.
11.4	4370
15.2	6210
19.15	8070
22.3	9530
25.66	10900
28.80	12040

Now taking $N = 0.0217$ for $m = 40$, we have

$$H = H' - NI = H' - KB$$

and $K = N/4\pi$, since we may neglect H in comparison with B. We get, therefore,

$$K = 0.00173,$$

and may now calculate H and the other quantities from the B of the above table. This gives us :

B.	$\Delta H = KB$.	H.	μ.	I.	κ.
4370	7.55	3.85	1135	348	90
6210	10.73	4.47	1390	493	110
8070	13.94	5.21	1548	641	123
9530	16.47	5.83	1634	758	130
10900	18.85	6.81	1600	865	127
12040	20.82	7.97	1520	960	120

We chose the value of N as would correspond to the ballistic step method. Had we, however, used the method of reversals with a solenoid wound on a pasteboard tube, or a split brass tube, then the ballistic throws observed would have been a little more than twice as great as those we found. If we take them as exactly twice as great, and if we assume that the time-constant of the solenoid is the same as for the short solenoid used in the earlier half of this work, then we should have

$$N = 0.0206 \qquad K = 0.00164$$

and the calculated values of the demagnetizing fields, the resultant fields, and the permeabilities would be :

ΔH	H	μ
7.17	4.23	1030
10.20	5.00	1240
13.22	5.93	1360
15.60	6.70	1420
17.90	7.76	1410
19.70	9.10	1320

This shows again how greatly different results obtained by step and reversal methods can be, if the observations are not properly corrected by using the appropriate N.

Distribution of Magnetic Induction.

In our theoretical discussion of the shape of the N-curves we found, page 197, that we might expect that the magnetization is much nearer uniformity when the applied field H' is quite small, than it is in the region of large susceptibility. Now several articles have been published on the distribution of magnetic induction in iron rods,[19] but the magnetizing fields which these writers used were of much greater strength than are necessary in order to investigate this particular question. However, Benedicks [20] found a very neat inverse relation between the susceptibility κ and the pole-distance in a short bar magnet. This is very clearly shown by Figure 21, which has been reproduced from his article. The curve called "Distance des Poles" has the ordinates l/L, where L = actual length of the bar magnet, and l = distance between poles, the method of determining l being based on the formula

$$\frac{l}{L} = \frac{I_{\text{mean}}}{I_{\text{max.}}},$$

[19] Phil. Mag., (5), **46**, 478–494 (1898), "On the Distribution of Magnetic Induction in Straight Iron Rods," J. W. L. Gill; Phil. Mag., (5), **48**, 262–271 (1889), "On the Distribution of Magnetic Induction in a Long Iron Bar," C. G. Lamb.

[20] Journ. de Physique, (4), **1**, 302–307 (1902), "Études sur la Distance des Pôles des Aimants"; Bihang Svenska Vet.-Akad. Handlingar, **27**, (1) No. 5, 23 pp. (1902), "Untersuchungen über den Polabstand Magnetischer Zylinder."

in which the I_{mean} is the magnetization as determined magnetometrically, and the $I_{\text{max.}}$ is found from the B as determined ballistically at the centre of the rod in the usual way. For this rod $\mathfrak{m} = 300$. The abscissae represent H', the magnetic field applied from without. Similar curves had also been previously published by Dr. L. Holborn,[21] only the susceptibilities were taken directly from the unsheared magnetization curve of a short cylinder.

Although these experiments of Holborn and Benedicks practically prove the increased uniformity of magnetization for low fields, it is perhaps a better plan to settle this point by a more direct method. It was therefore thought that it might be of interest to compare the

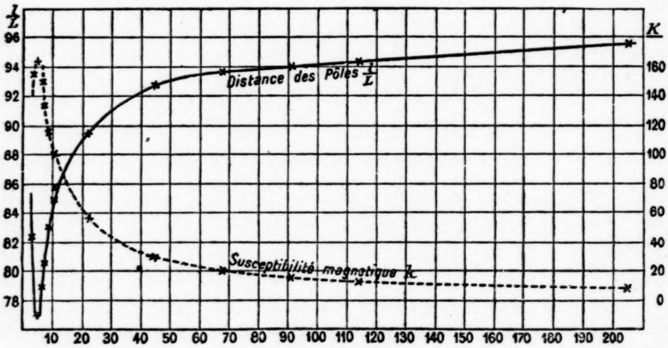

FIGURE 21.

Benedicks's curves, showing variation of the pole-distance ratio and the susceptibility in an iron rod. The abscissae give the field H′ in c. g. s. units.

actual magnetic induction which passes through various cross-sections of some of our iron rods, for practically the whole range of magnetization from zero to saturation. To do this one might use a secondary search-coil, fitting loosely around the iron rod, which can be suddenly displaced along the rod by any desired distance. This would require two observers; but it could not be used conveniently in this work since the rods in which the magnetic induction was tested were 1.905 cms. in diameter, and the inner diameter of the brass tube around which the solenoid coils were wound was not much larger. Another method would be to wind coils around different parts of the rod and get the actual induction passing through each coil. This would do

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[21] Sitzber. Akad. d. Wiss., Berlin, **1**, 159–162 (1898).

well enough for the lower intensities of H' but would be an exceedingly insensitive method to use when the field H' is very high, since then the induction is nearly constant along the bar except at the very ends, so that the experimental error might easily be even greater than the actual difference in the magnetic induction between the central part of the rod and any other part. The best method seems to be to read the reversal method ballistic throw from a coil wound directly over the middle of the rod, and then, connecting any other coil, wound around the rod nearer the end, in series with the central coil but in opposition to it, observe the ballistic throw due to the difference in the flux

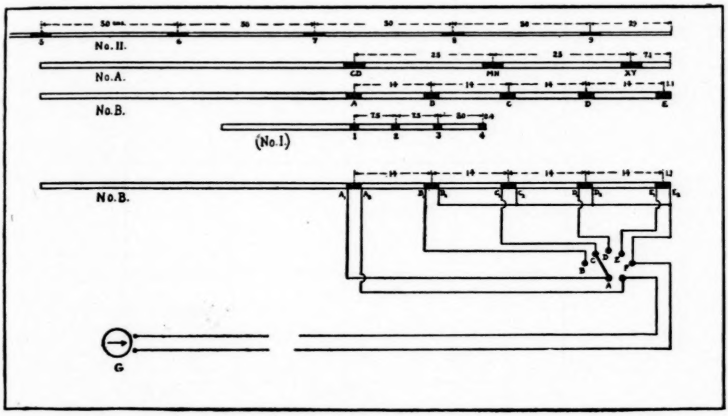

FIGURE 22.

Diagram showing arrangement of secondary coil and switch-board used in the work on the distribution of magnetic induction along an iron rod.

through the two coils. This was the idea adopted. Figure 22 shows diagrammatically the arrangement of the coils in one of the four different cases which were tried; the others were similarly arranged. The positions of all the secondary coils are shown in the diagrams drawn to scale and marked with the distances between the centres of the coils.

All the ends of the coils were led into small mercury cups in a small switchboard. The extremities B_2, C_2, D_2, E_2, and one terminal of the ballistic galvanometer were all dipped into cup F. If now the copper connector is placed in the position $A\ C$ as shown, then the ballistic throw observed on reversing the current in the primary solenoid is that

due to those lines of magnetic induction which thread through the centrally placed coil $A_1 A_2$ and do not also pass through the coil $C_1 C_2$, provided we neglect the lengths $A_1 A_2$ and $C_1 C_2$ of the secondary coils in comparison with the distance $A_1 C_1$ between the two coils. In other words, the ballistic throw measures the magnetic leakage between the coils which are connected in opposition. When the connector is placed across from A to F, then we get simply the throw

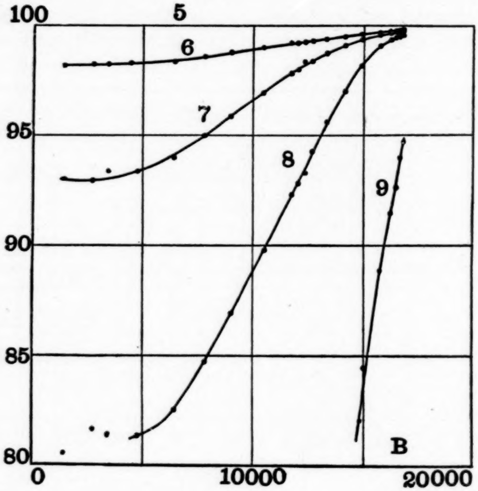

FIGURE 23.

Curves showing variations in the distribution of magnetic induction in rod No. II. D = 1.905 cms. and $m = 240$. The ordinate-axis represents percentage of magnetic induction.

due to the whole magnetic flux of induction through the central coil $A_1 A_2$ in precisely the manner which was used in all of the preceding work on magnetization curves for different m's.

In this work on the distribution of the magnetic induction the extra resistance which had to be thrown into the galvanometer circuit by means of the resistance box R′ in order to regulate the throw, varied greatly. For a connection like that shown in the figure usually no extra resistance was needed; in fact for low as well as for high magnetizing fields the magnetic induction approaches uniformity, so that in either case the ballistic throw is very low. Thus while in a certain

case $\mathfrak{m} = 25$, and $B = 21120$, the extra resistance R' had to be made as high as 10,000 ohms in order to keep the throw for the central coil alone from exceeding the length of the scale, yet when the coil nearest to the central one was connected in opposition to it, only a weak deflection was obtained with no extra resistance in the galvanometer circuit.

The curves which are shown represent four different rods, all having the largest diameter used, 1.905 cms., but two of these had the same length, the $\mathfrak{m}$ being $= 60$, so that for these rods the results are combined in one figure. The data for these four rods are as follows:

TABLE XIX.

Bessemer Rod $D = 1.905$.	$\mathfrak{m}$.	Turns per Coil.	Length of each Coil.	Range of H'.	Range of B.	Maximum Battery Voltage.
No. II.	240	50	3.7 cms.	0.77– 63.0	1620–16800	20
No. B.	60	50	2.3 "	0.50– 66.8	84–16980	20
No. A.	60	50	3.6 "	0.25– 67.7	25–16800	20
(No. I)	25	110	1.3 "	3.7 –440.0	550–21120	40

Bessemer Rod.	Length of Solenoid.	No. of Coils.	Distances between Coils in Cms.
No. II.	485.3 cms.	5	50, 50, 50, 50; 29 to end.
No. B.	" "	5	14, 14, 14, 14; 1.1 to end.
No. A.	" "	3	25, 25; 7.1 to end.
(No. I)	107.2 "	4	7.5, 7.4, 8.0; 0.4 to end.

The coils are designated as follows, beginning with the central one:

No. II. 5–6–7–8–9. No. B. A–B–C–D–E.
No. A. CD–MN–XY. (No. I) 1–2–3–4.

The results are given graphically by Figures 23, 24, and 25 in this way: The induction B in the middle part of the rod, as found from reversing the current in the solenoid while only the central coil is included in the galvanometer circuit, is plotted horizontally; while the ordinates give the ratio of the corresponding inductions in the parts of

the rod surrounded by the other coils, to the induction at the centre. Thus, suppose for a given constant H' we had obtained throws corresponding to the central coil alone, and also for this coil when connected in opposition to every one of the other coils in turn. In an actual case we had for Rod B: $H' = 59.5$, the induction for the central coil was $B = 16{,}560$, leakage between CD and MN was 630, and between CD and XY 7910, lines of induction per unit cross-section. From these results we get for the actual magnetic induction through MN

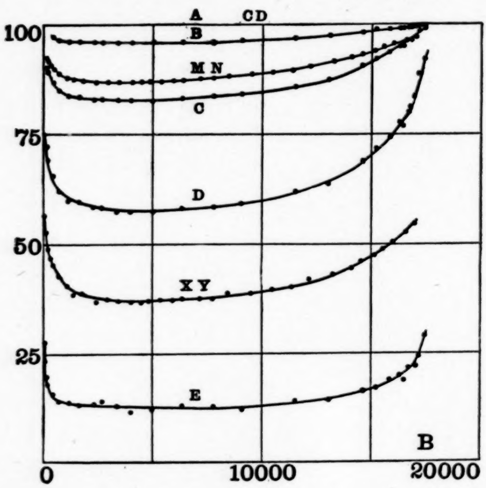

FIGURE 24.

Curves showing variations in the distribution of magnetic induction in rods No. A and No. B. D = 1.905 cms. and m = 60.

15,920 lines, and through XY 8650 lines. Now, denoting the B through the central coil at any time by 100 per cent, we shall have 96.3 per cent of this induction passing also through the coil MN, and 52.3 per cent through XY. These two numbers are therefore plotted against $B = 16{,}560$. Figures 23, 24, and 25 exhibit all the observations taken. The slight zigzag arrangement of the points is due to the fact that the current did not stay quite constant during the time of observing the throws from all the coils on a rod. All the rods have been referred to previously by the same designations, except (No. I), which is merely one of the end-pieces cut from the long rod

No. I mentioned before. The crossing of the curves for coils MN and C at a high induction is merely another instance of the great difference in magnetic quality of Rods A and B (or Rods I and II) which was already noticed in the magnetization curves of Figure 15.

From the curves in Figures 24 and 25 we see that for low fields there is quite an increase in the induction for coils not at the middle of the rod as compared with the induction through the central coil. This means that for these low fields the magnetization is more nearly

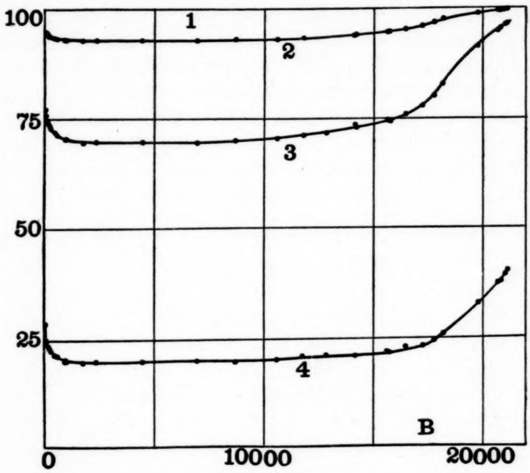

FIGURE 25.

Curves showing variations in the distribution of magnetic induction in rod (No. I). D = 1.905 cms. and m = 25.

uniform. The range in H' for which the sharp upward bend of these curves occurs is precisely the same range for which the susceptibility changes most rapidly and is from $H' = 0$ to about $H' = 5$. After this we have quite a long interval for which the susceptibility is high, and the magnetization furthest removed from uniformity; here the curves showing percentage of induction as compared to that through the middle coils have their minimum and run along very nearly parallel to the B-axis. However, as the induction through the middle of the rod increases past $B = 10{,}000$, all the curves begin to rise, slowly at first, then more rapidly. This indicates that the susceptibility is again

decreasing, and that the magnetization is becoming gradually more and more uniform. At about $B = 17{,}000$ the curves rise the fastest, showing that the middle portions of the rod are very nearly saturated and take up more magnetization only very slowly, while for the coils nearer the end the magnetization is still rapidly increasing. Figure 25, for the short rod ($\mathfrak{m} = 25$), shows that after B is about 20,000 under the middle coil, the curves all have points of inflection and now approach the ordinate 100 per cent asymptotically. If we now consider Figure 23, for the very long rod ($\mathfrak{m} = 240$), we see that here we have a case of the magnetization being always very much nearer uniformity, so that the curves for coils 6, 7, and 8 are already in the asymptotic stage for $B = 15{,}000$ under the coil 5, and the points of inflection are near $B = 10{,}000$. When $B = 15{,}000$, the curve for the coil 9, nearest the end of the rod, shows a tremendous upward shoot from a long horizontal course near the ordinate 50 per cent. Since the figure only gives the observations in the range of percentages from 80 to 100, it might be well to give the missing values here :

B in Coil 5.	Percentage : $\frac{B_9}{B_5}$.	B in Coil 5.	Percentage : $\frac{B_9}{B_5}$.
2720	50.3	11730	65.30
3420	52.5	12030	65.67
4720	51.1	12330	67.00
6420	51.2	12680	68.70
7800	54.5	13330	71.90
9000	56.5	14130	76.66
10470	60.8	. . .	. . .

In the case of the long rod the lowest fields used were still too high to show a rise in the curves, corresponding to increased uniformity of magnetization, as is seen in the other two figures.

The results show that near the middle of a rod the induction is practically the same for quite a little range, especially if the rod is fairly long. Thus the curve 6 in Figure 23 shows that in the rod of length about 458 cms. and $\mathfrak{m} = 240$, the induction for a distance 50 cms. from the middle of the rod is always within about 2 per cent of the induction at the middle portion. And curve B in Figure 24 proves the induction at 14 cms. from the middle of the rod of length about

114 cms. and $\mathfrak{m} = 60$ to be always within about 4 per cent of the central induction. These facts justify the use of a secondary coil several cms. in length, provided the $\mathfrak{m}$ of the rod is not too small.

The conclusion to be reached from the work on the induction distribution is that for low field-intensities, as well as for high ones, the magnetization of the iron rod is much more nearly uniform than it is in a long interval corresponding to rather high susceptibilities.

Discussion of Results obtained.

When we look over the tables we readily see a number of interesting things. It is apparent that in general different methods or even different experimental conditions will give different normal curves, and hence different susceptibility curves. A striking result, and one which was obtained entirely unexpectedly, is that in the long solenoid, which was wound on a thick brass tube, the method of reversals agrees very closely indeed with the step-by-step method. This may in fact turn out to be quite a useful observation, for it points to the probability of getting values for the susceptibility of some kind of iron in the form of a short rod, which conform very closely to the ideal definition of susceptibility, which requires slow, continuous increase of the magnetizing field. Thus by winding our solenoid on very thick brass tubes, a large E. M. F. from a storage battery may be suddenly turned on, without giving almost instantaneously the full value of the magnetizing field within, on account of the eddy currents in the brass tube acting as a sort of "brake."

The most important results described in this paper about the demagnetizing factor N for cylindrical iron rods are the following:

(1) The demagnetizing factor is not a constant, but shows two opposite curvatures, when plotted as abscissa-differences ($H_i = NI$) on the I vs. H_i plane; while for the highest values of I it falls to about $\frac{1}{7}$ or $\frac{1}{8}$ of its value for unsaturated I's.

(2) For values of B less than 10,000 the N is practically constant.

(3) Using a solenoid made of wire wound on a non-metallic tube, or a split brass tube, the reversal method gives values for N considerably lower than the step-by-step method.

(4) If the magnetizing solenoid is wound on a thick brass tube, the reversal and step methods practically agree, and values of N derived from curves taken in this way are regarded as the most desirable for scientific purposes, as they will give most accurate values for the susceptibility or the permeability of the iron.

(5) The demagnetizing factors are largest for thin rods. The differ-

ences between the corresponding N's for a rod of 0.3175 cm. diameter and one of 1.905 cms. diameter range from 10 to 16 per cent, both sets of values being taken to conform to the conditions stated in (4).

(6) Most of the rods used in this work have their N's in the range of practical constancy considerably smaller than the values given by Du Bois, but as the diameters of the rods decrease, a very close approach to Du Bois's values is obtained.

(7) The magnetization is furthest away from uniformity in the region of highest susceptibilities, and becomes more uniform for very low as well as for very high applied fields.

In conclusion it is my pleasant duty and privilege to thank Professor B. O. Peirce for suggesting this research and for his constant interest in the work throughout the year. I also desire to state that the astaticised galvanometer system is due to the skill of Mr. John Coulson, Professor Peirce's assistant; and that the construction of the magnetizing solenoid was most successfully carried out by Mr. Thompson, the mechanic of the Jefferson Physical Laboratory.

Literature on the Demagnetizing Factor.

J. A. Ewing: "Experimental Researches in Magnetism," Phil. Trans., **176**, 523–640 (1885). (Plate 57, Fig. 3.)

A. Tanakadaté: "Mean Intensity of Magnetization of Soft Iron Bars of Various Lengths in a Uniform Magnetic Field," Phil. Mag., (5), **26**, 450–456 (1888).

H. E. J. G. Du Bois: "Zur mathematischen Theorie des Ferromagnetismus," Wied. Ann., **46**, 485–499 (1892) (also in "Magnetische Kreise in Theorie und Praxis," Berlin, 1894, p. 37).

C. R. Mann: "Ueber Entmagnetisirungsfaktoren kreiscylindrischer Stäbe," Dissertation Berlin, 1895; "Demagnetization Factors for Cylindrical Rods," Phys. Rev., **3**, 359–369 (1896).

Ascoli e Lori: "Sopra il fattore smagnetizzante nei cilindri di ferro," Rendic. R. Acad. d. Lincei, **3**:**2**, 190 (1894); Ascoli: "Sul fattore smagnetizzante nei fasci e nei cilindri di ferro," Rendic. R. Acad. d. Lincei, **6**:**2**, 129 (1897).

Carl Benedicks: "Ueber die Entmagnetisirungsfaktoren kreiscylindrischer Stäbe," Wied. Ann., **6**, 726–761 (1901); "Sur les facteurs démagnetisants des cylindres," Bihang Svenska Vet.-Akad. Handlingar, **27**, (1), No. 4, 14 pp. (1902).

H. E. J. G. Du Bois: "Entmagnetisirungsfaktoren kreiscylindrischer Stäbe," Wied. Ann., (4), **7**, 942–943 (1902).

Literature on Related Subjects.

Eduard Riecke: "Zur Lehre von den Polen eines Stabmagnetes," Wied. Ann., **8**, 299–325 (1879).

C. Baur: "Neue Untersuchungen über den Magnetismus," Wied. Ann., **11**, 394–413 (1880).

F. Kohlrausch: "Ueber den Polabstand, den Inductions- und Temperatur-coëfficient eines Magnetes und über die Bestimmung von Trägheitsmomenten durch Bifilarsuspension," Wied. Ann., **22**, 411–424 (1884).

Lord Rayleigh: "I. On the Energy of Magnetized Iron," Phil. Mag., (5), **22**, 175–183 (1886); "Notes on Electricity and Magnetism. — III. On the Behavior of Iron and Steel under the Operation of Feeble Magnetic Forces," Phil. Mag., (5), **23**, 225–245 (1887).

H. E. J. G. Du Bois: "On Magnetization in Strong Fields at Different Temperatures," Phil. Mag., (5), **29**, 293–306 (1890).

O. Grotrian: "Der Magnetismus eiserner Hohl- und Voll-cylinder," Wied. Ann., **50**, 705–741 (1893); "Zur Magnetisirung von eisernen Cylindern," Wied. Ann., **52**, 735–748 (1894); also, **54**, 452–475 (1894).

Gustav Rössler: "Untersuchungen über die Magnetisirung des Eisens durch sehr kleine und durch sehr grosse Kräfte," Elekt. ZS, **14**, 97–99, 114–116, 133–134, 149–151, 161–163 (1893).

H. E. J. G. Du Bois: Note on Rössler's article, Elekt. ZS, **14**, 208 (1893).

P. Culmann: Note on Rössler's work, Elekt. ZS., **14**, 345 (1893); "Ueber die Gültigkeit eines von Kirchhoff in der Theorie des Electromagnetismus aufgestellten Satzes," Wied. Ann., **48**, 380–383 (1893).

J. L. W. Gill: "On the Distribution of Magnetic Induction in Straight Iron Rods," Phil. Mag., (5), **46**, 478–494 (1898).

Dr. L. Holborn: "Ueber die Vertheilung des inducirten Magnetismus in Cylindern," Sitzungsberichte d. Akad. d. Wiss. zu Berlin, **1**, 159–168 (1898).

F. Kirstaedter: "Zur Magnetisirung eiserner Hohl- und Vollringe," Wied. Ann., **65**, 72–85 (1898).

C. G. Lamb: "On the Distribution of Magnetic Induction in a Long Iron Bar," Phil. Mag., (5), **48**, 262–271 (1899).

Carl Benedicks: "Untersuchungen über den Polabstand magnetischer Zylinder," Bihang Svenska Vet.-Akad. Handlingar, **27**, (1), No. 5, 23 pp. (1902); "Etudes sur la distance des pôles des aimants." Journ. de Phys., (4) **1**, 302–307 (1902).

G. T. C. Searle and T. G. Bedford: "The Measurement of Magnetic Hysteresis," Phil. Trans., A **198**, 33–104 (1902); Abstract of this paper in Proc. Roy. Soc., **68**, 348–352 (1901).

H. Meldau: "Magnetisirung eiserner Zylinder," Phys. ZS. **4**, 479–480 (1903).

Raymond Jouaust: "Les phenomènes de viscosité magnétique dans les aciers doux industriels et leur influence sur les methodes de mesure," Comp. Rend., **139**, 272–274 (1904).

Franz Rücker: "Beiträge zur Kenntnis der stufenweisen und stetigen Magnetisirung," Dissertation, Halle, 1905, 106 pp., 20 plates; Elekt. ZS., **26**, 904–905, 979 (1905).

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